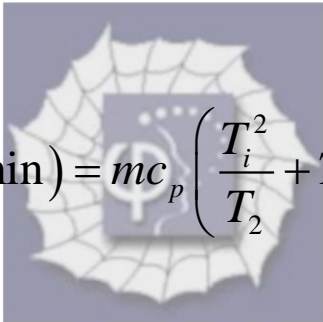


269.

**Problem 10.14 (Zemansky)**

*Two identical bodies of constant heat capacity are at the same initial temperature  $T_i$ . A refrigerator operates between these two bodies until one body is cooled to temperature  $T_2$ . If the bodies remain at constant pressure and undergo no change of phase, we have to show that the minimum amount of work needed to do this is*


$$W(\min) = mc_p \left( \frac{T_i^2}{T_2} + T_2 - 2T_i \right).$$

**Solution:**

We are given that we have two identical bodies each of mass  $m$  at temperature  $T_i$ . A refrigerator operates between these two bodies until one body is cooled to temperature  $T_2$ . Let the amount of work done on the refrigerator in cooling of body 2 from temperature  $T_i$  to  $T_2$  be  $W$ . The quantity of heat extracted out of body 2 will be

$$Q = mc_p (T_i - T_2).$$

And the amount of heat delivered by the refrigerator to the body 1 will be

$$Q' = Q + W.$$

Temperature of body 1 will have got raised from  $T_i$  to  $T'$  such that

$$\begin{aligned} mc_p (T' - T_i) &= Q + W \\ &= mc_p (T_i - T_2) + W. \end{aligned}$$

From this equation, we find

$$T' = 2T_i - T_2 + \frac{W}{mc_p}.$$

We next calculate the change in entropy of the body 1 and that of the body 2.

$$\Delta S_1 = mc_p \ln \left( \frac{T'}{T_1} \right) = mc_p \ln \left( \frac{2T_i - T_2 + \frac{W}{mc_p}}{T_i} \right).$$

And

$$\Delta S_2 = mc_p \ln \left( \frac{T_2}{T_i} \right).$$

The total change of entropy in the process will be

$$\Delta S = \Delta S_1 + \Delta S_2 = mc_p \ln \left( \frac{\left( 2T_i - T_2 + \frac{W}{mc_p} \right) \times T_2}{T_i^2} \right).$$

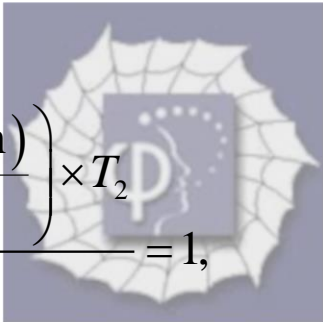
From the entropy principle

$$\Delta S \geq 0.$$

Minimum amount of work will be done on the refrigerator in the process for which

$$\Delta S = 0.$$

That is

$$\frac{\left( 2T_i - T_2 + \frac{W(\min)}{mc_p} \right) \times T_2}{T_i^2} = 1,$$


or

$$W(\min) = mc_p \left( \frac{T_i^2}{T_2} + T_2 - 2T_i \right).$$