267.

<u>Problem 10.12 (Zemansky)</u>

A body of finite mass is originally at a temperature T_1*, which is higher than that of a reservoir at the temperature* T_2*. Suppose an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from* T_1 *to* T_2*, thus extracting heat Q from the body. If the engine does work W, then it will reject heat Q-W to the reservoir at* T_2*. Applying the entropy principle, we have to prove that the maximum work obtainable from the engine is*

$$W(\text{max}) = Q - T_2(S_1 - S_2),$$

where $S_1 - S_2$ *is the entropy decrease of the body.*

<u>Solution:</u>

The entropy principle is that the change of entropy of the universe as a result of any kind of process is greater than or equal to zero. That is

$$\Delta S(\text{universe}) \geq 0.$$

It is given that a body of finite mass is originally at a temperature T_1, which is higher than that of a reservoir at the temperature T_2. An engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2, thus extracting heat Q from the body. If the engine does work W, then it will reject heat Q-W to the reservoir at T_2. Let the change in entropy of the body be

$$S_2 - S_1.$$

As we are considering that the body is connected to a reversible cyclic engine, the change in entropy of the engine is zero.

As an amount of heat Q-W is added to the reservoir, which is at temperature T_2, the change in entropy of the reservoir will be

$$\frac{Q-W}{T_2}.$$

The change in entropy of the universe will be

$$\Delta S = S_2 - S_1 + \frac{Q-W}{T_2}.$$

According to the entropy principle

$$S_2 - S_1 + \frac{Q - W}{T_2} \geq 0.$$

Or

$$W \leq Q + T_2\left(S_2 - S_1\right).$$

Therefore,

$$W\left(\text{max}\right) = Q - T_2\left(S_1 - S_2\right).$$