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<u>Problem 10.11 (Zemansky)</u>

A mass m of water at T_1 is isobarically and adiabatically mixed with equal mass of water at T_2. We have to show that the entropy change of the universe is

$$2mc_p \ln \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}},$$

and prove that this is positive.

<u>Solution:</u>

As equal amounts of water equal to m at temperatures T_1 and T_2 are mixed. It is given that the process is carried out isobarically and adiabatically. It means that the exchange takes place between the two equal mass water samples only and there is no exchange of heat with the surroundings.

Therefore, when thermal equilibrium is reached the temperature of water will become $T_e = (T_1 + T_2)/2$. The thermodynamic process is irreversible.

We will calculate the change in entropy by using reversible thermodynamic processes in which the samples of water at temperatures T_1 and T_2 reach the equilibrium temperature T_e gradually and the entropy change can be calculated using the result

$$dS = \frac{dQ}{T} = \frac{mc_p dT}{T}.$$

We have

$$\Delta S = \Delta S_1 + \Delta S_2 = mc_p \left(\int_{T_1}^{T_e} \frac{dT}{T} + \int_{T_2}^{T_e} \frac{dT}{T} \right) = mc_p \left(2\ln T_e - \ln\left(T_1 T_2\right) \right)$$

$$= 2mc_p \ln\left(\frac{T_1 + T_2/2}{\sqrt{T_1 T_2}} \right).$$

We will next show that

$$\frac{\left(T_1 + T_2\right)^2}{4T_1T_2} > 1.$$

Or

$$\frac{\left(T_1 + T_2\right)^2}{4T_1T_2} - 1 > 0.$$

Note that

$$\frac{\left(T_1 + T_2\right)^2}{4T_1T_2} - 1 = \frac{\left(T_1 - T_2\right)^2}{4T_1T_2} > 0.$$

Therefore, $\Delta S > 0$, as is required by the second law of thermodynamics for an irreversible process.