265.

Problem 26.45 (RHK)

A 12.6-g *ice cube at* $-10.0^0\mathrm{C}$ *is placed in a lake whose temperature is* $+15.0^0\mathrm{C}$. *We have to calculate the change in entropy of the system as the ice cube comes to thermal equilibrium with the lake.*

Solution:

In the process of reaching thermal equilibrium the 12.6 g of ice at $-10.0^0\mathrm{C}$ will first get heated to $0^0\mathrm{C}$, then will melt by acquiring heat of fusion, and next will get heated to $+15.0^0\mathrm{C}$, the temperature of the lake. We can assume the lake to be an infinite heat reservoir at $+15.0^0\mathrm{C}$. The total heat for the sequence of changes in the ice cube at $-10.0^0\mathrm{C}$ to its becoming water at $+15.0^0\mathrm{C}$ will be supplied by the lake, which, as already mentioned, functions like a heat reservoir at a constant temperature. We will use the following data in calculating the total change of entropy of the system.

Heat capacity of water, $c_w = 4190$ J kg^{-1} K^{-1}, heat capacity of ice, $c_i = 2220$ J kg^{-1} K^{-1},

heat of fusion, $L_{fusion} = 333$ kJ kg^{-1}.

(i) The change in entropy of the ice when the ice cube gets heated from -10.0^0C to 0^0C,

$$\Delta S_1 = \int_{263}^{273} \frac{m_{ice}c_V}{T} dT = 12.6\times10^{-3}\times2220\times\int_{263}^{273}\frac{dT}{T} \text{ J K}^{-1}$$

$$= 1.0438 \text{ J K}^{-1}.$$

(ii) The change in entropy of the ice in melting of 12.6 g of ice at 0^0C,

$$\Delta S_2 = \frac{12.6\times10^{-3}\times333\times10^3}{273} \text{ J K}^{-1}$$

$$= 15.369 \text{ J K}^{-1}.$$

(iii) The change in entropy of the 12.6g of water when it gets heated from 0^0C to $+15.0^0\text{C}$, the temperature of the lake,

$$\Delta S_3 = \int_{273}^{288} \frac{m_i c_w dT}{T} = 12.6\times10^{-3}\times4190\times\ln\left(\frac{288}{273}\right) \text{ J K}^{-1}$$

$$= 2.8238 \text{ J K}^{-1}.$$

(iv) The total amount of heat supplied by the lake in the processes (i), (ii) and (iii) is

$$Q = 12.6\times10^{-3}\left(2220\times\left(0-\left(-10.0\right)\right)+333000+4190\times\left(15-0\right)\right)\ \mathrm{J}$$

$$= 5267.4\ \mathrm{J}.$$

The change of entropy of the lake will be

$$\Delta S_4 = -\frac{5267.4}{288}\ \mathrm{J\ K^{-1}} = -18.2896\ \mathrm{J\ K^{-1}}.$$

Therefore, the total change of entropy of the system will be

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 = \left(1.0438 + 15.369 + 2.8238 - 18.2896\right)\mathrm{J/K}$$

$$= 0.947\ \mathrm{J\ K^{-1}}.$$