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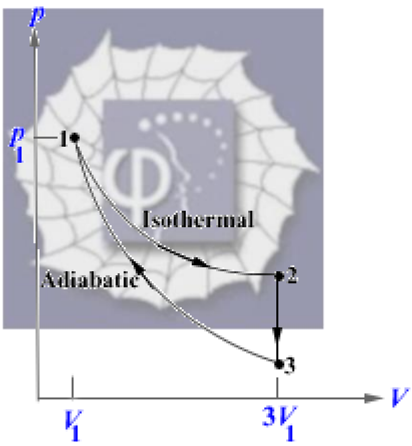
Problem 26.40 (RHK)

One mole of an ideal diatomic gas is caused to pass through the cycle shown on the pV diagram, where $V_2 = 3V_1$. *We have to determine in terms of* p_1, V_1, T_1 *and R:* (a) $p_2, p_{3,}$ *and* T_3, *and* (b) W, Q, ΔE_{int}, and ΔS *for all the three processes.*



Solution:

We are considering thermodynamic cyclic processes of one mole of a diatomic ideal gas. For a diatomic gas the ratio of specific heats $\gamma = 7/5$.

Three states of the gas marked by labels 1,2, and 3 are connected by thermodynamic processes shown in the *pV* diagram. Pressure, volume, and temperature of the gas at the point 1 are

$p = p_1,$

$V = V_1,$

and

$T = T_1$

Using the ideal gas equation of state, we note that the values of volume, pressure, and temperature at 2 are

$V = 3V_1,$

$p = \frac{1}{3} p_1,$

and

$T = T_1 \, .$

Volume at 3 is

$V = 3V_1.$

States 1 and 3 are connected by an adiabatic process. Therefore,

$p_3 \left(3V_1\right)^{\gamma} = p_1 V_1^{\gamma},$

or

$p_3 = \frac{1}{3^{\gamma}} p_1.$

And, $T_3 = \frac{1}{3^{\gamma - 1}} T_1.$

We have already noted that for a diatomic ideal gas

$\gamma = 7/5.$

Process connecting the states 1 and 3 is isothermal.

Therefore,

$E_{\text{int},2} - E_{\text{int},1} = 0.$

Work done on the gas will be

$$W_{12} = -\int_{V_1}^{V_2} p dV = -\int_{V_1}^{V_2} \frac{RT_1}{V} dV = -RT_1 \ln\left(\frac{V_2}{V_1}\right) = -RT_1 \ln 3.$$

And heat absorbed by the gas during this part of the cycle will be

$Q_{12} = -\left|W_{12}\right| = RT_1 \ln 3.$

Change in entropy of the gas between the states 2 and 1 will be

$$\Delta S_{12} = S_2 - S_1 = \frac{Q_{12}}{T_1} = R \ln 3.$$

Process joining states 3 and 2 takes place at constant volume. Therefore, no work is done on the gas during this process. Heat absorbed by the gas, Q_{23}, is equal to the change in internal energy between these two states.

$$Q_{23} = E_{\text{int},3} - E_{\text{int},2} = (\gamma - 1)^{-1} R(T_3 - T_2) = (\gamma - 1)^{-1} RT_1\left(\frac{1}{3^{\gamma-1}} - 1\right)$$

$$= -0.88RT_1.$$

Change in entropy

$$\Delta S_{23} = S_3 - S_2 = \int_{T_2}^{T_3} \frac{C_V dT}{T} = \frac{R}{(\gamma - 1)} \ln\left(\frac{T_3}{T_2}\right) = \frac{R}{(\gamma - 1)} \ln\left(\frac{1}{3^{\gamma-1}}\right)$$

$$= -R\ln 3.$$

States 1 and 3 are connected by an adiabatic process. In an adiabatic process there is no exchange of heat energy. That is $Q_{31} = 0$. Therefore,

$$\Delta S_{31} = S_3 - S_1 = 0.$$

Work done on the gas during the adiabatic process will be

$$W_{31} = \frac{1}{(\gamma - 1)}(p_1V_1 - p_3V_3) = \frac{1}{(\gamma - 1)} p_1V_1\left(1 - \frac{1}{3^{\gamma-1}}\right).$$

And,

$$E_{\text{int},1} - E_{\text{int},3} = \frac{RT_1}{(\gamma - 1)} \times \left(1 - \frac{1}{3^{\gamma-1}}\right) = 0.88RT_1.$$