Problem 26.37 (RHK)

An ideal gas undergoes an isothermal expansion at 77°C, increasing its volume from 1.3 to 3.4 L. The entropy change of the gas is 24 J K⁻¹. We have to find the amount of gas present in moles.

Solution:

Let us recall that in an isothermal process there is no change in the internal energy. Therefore, the work done by the gas in isothermal expansion from initial volume V_i to final volume V_f will be equal in magnitude to the amount of heat absorbed. The work done by the gas (*n* moles) in isothermal expansion from volume V_i to volume V_f will be

$$|W| = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right).$$

Therefore, the heat absorbed, Q, in isothermal expansion of n moles of an ideal gas will be

$$Q = nRT \ln\left(\frac{V_f}{V_i}\right).$$

The change in entropy of *n* moles of an in isothermal expansion will therefore be

$$S_f - S_i = \frac{Q}{T} = nR \ln\left(\frac{V_f}{V_i}\right).$$

Data of the problem are

$$V_f = 3.4 \text{ L},$$

 $V_i = 1.3 \text{ L},$
and
 $\Delta S = 24 \text{ J K}^{-1}.$
We have
 $n \times 8.31 \ln \left(\frac{3.4}{1.3}\right) \text{ J mol}^{-1} \text{ K}^{-1} = 2.4 \text{ J K}^{-1}$

or

$$n = \frac{24}{8.31 \times 0.961}$$
 mol. = 3.0 mol.