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Problem 26.33 (RHK)

At very low temperatures, the molar specific heat of many solids is (approximately) proportional to T^3*; that is* $C_V = AT^3$*, where A depends on the particular substance. For aluminium,* $A = 3.15\times10^{-5}$ J mol^{-1} K^{-4}. *We have to find the entropy change of* 4.8 mol *of aluminium when its temperature is raised from* 5.0 *to* 10 K.

Solution:

Let us assume that change of temperature of the sample of aluminium from 5.0 to 10 K is by a quasi-static process. In this situation the entropy change can be calculated from the specific heat at constant volume, which varies with temperature as

$C_V = AT^3$. The amount of heat absorbed by the aluminium sample will be

$dQ(T) = n\times C_V(T)\times dT$,

where n is the amount of the substance in moles. The change in entropy when the temperature of the sample is raised from T_i to T_f will be given by the integral

$$\Delta S = \int_{T_i}^{T_f} \frac{dQ}{T} = nA \int_{T_i}^{T_f} \frac{T^3 dT}{T} = \frac{nA\left(T_f^3 - T_i^3\right)}{3}.$$

Data of the problem are

$T_i = 5.0$ K,

$T_f = 10.0$ K,

and

$A = 3.15 \times 10^{-5}$ J mol^{-1} K^{-4}.

Therefore, the change in entropy of the sample of aluminium when heated from 5.0 K to 10.0 K will be

$$\Delta S = 4.8 \times 3.15 \times 10^{-5} \times \frac{\left(10^3 - 5^3\right)}{3} \text{ J K}^{-1} = 0.044 \text{ J K}^{-1}.$$