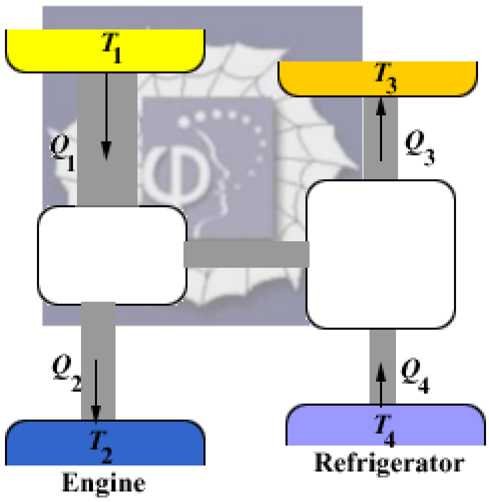
255.

<u>Problem 26.23 (RHK)</u>

A Carnot engine works between temperatures T_1 *and* T_2. *It drives a Carnot refrigerator that works between two different temperatures* T_3 *and* T_4. *We have to find the ratio* $|Q_3|/|Q_1|$ *in terms of the four temperatures.*



<u>Solution:</u>

Let W be the amount of wok which becomes available per cycle of the Carnot engine operating between temperatures T_1 and T_2. Let $|Q_1|$ be the amount of heat absorbed by the engine from the heat reservoir at temperature T_1. From the second law of thermodynamics, we have

$$\frac{W}{|Q_1|} = \frac{T_1 - T_2}{T_1},$$

and therefore the amount of work output of the engine that can be used for running the refrigerator will be

$$W = |Q_1| \times \frac{T_1 - T_2}{T_1}.$$

From the second law of thermodynamics we get for the amount of heat extracted by the Carnot refrigerator from the heat reservoir at temperature T_4 and the heat discharged to the surroundings at temperature T_3 will be

$$\frac{|Q_4|}{|Q_3|} = \frac{T_4}{T_3},$$

and

$$\frac{|Q_3| - |Q_4|}{|Q_3|} = \frac{T_3 - T_4}{T_3}.$$

But

$|Q_3| - |Q_4| = W$, the work delivered by the Carnot engine to the Carnot refrigerator per cycle.

Therefore,

$$\frac{W}{|Q_3|} = \frac{T_3 - T_4}{T_3}.$$

And we get

$$\frac{|Q_1|}{|Q_3|} \times \frac{T_1 - T_2}{T_1} = \frac{T_3 - T_4}{T_3}.$$

By rearranging the above expression, we get

$$\frac{|Q_3|}{|Q_1|} = \frac{(1 - T_2/T_1)}{(1 - T_4/T_3)}.$$