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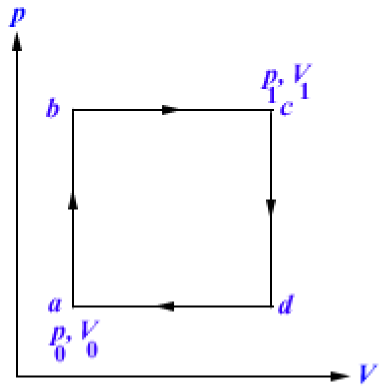
Problem 26.27 (RHK)

One mole of an ideal monatomic gas is used as the working substance of an engine that operates on the cycle shown in the figure. We have to calculate (a) *the work done by the engine per cycle;* (b) *the heat added per cycle during the expansion stroke abc; and* (c) *the engine efficiency.* (d) *We have to find the Carnot efficiency of an engine operating between the highest and lowest temperatures present in the cycle. We have to compare the Carnot efficiency with the efficiency calculated in* (c). *We may assume that* $p_i = 2p_0$, $V_i = 2V_0$, $p_0 = 1.01\times10^5\ \text{Pa}$, *and* $V_0 = 0.025\ \text{m}^3$.



Solution:

(a) and (b)

Data of the problem are

$p_a = p_0 = 1.01\times10^5$ Pa,

$V_a = V_0 = 2.25\times10^{-2}$ m^3,

$p_b = p_1 = 2p_0$,

$V_b = V_0$,

$p_c = p_1 = 2p_0$,

$V_c = V_1 = 2V_0$,

and

$p_d = p_0$,

$V_d = V_1 = 2V_0$.

The working substance of the engine is one mole of an ideal monatomic gas. Therefore, the ratio of its specific heats $\gamma = 5/3$ and $n = 1$.

The process *ab* takes place at constant volume. Therefore, during this part of the process no work is done on the gas and an amount of heat Q_{ab} which is equal to ΔE_{ab} is absorbed. That is

$$Q_{ab} = \Delta E_{ab} = \frac{3}{2} R\left(T_b - T_a\right) = \frac{3}{2}\left(p_b V_b - p_a V_a\right)$$
$$= \frac{3}{2}\left(2 p_0 V_0 - p_0 V_0\right)$$
$$= \frac{3}{2} p_0 V_0.$$

During the part *bc* of the cycle, amount of work W_{bc} is done on the gas and amount of heat Q_{bc} is absorbed by the gas. As the expansion of the gas takes place at constant pressure, we have

$$W_{bc} = -p_1\left(V_c - V_b\right) = -2 p_0 V_0.$$

And

$$\Delta E_{bc} = \frac{3}{2} R\left(T_c - T_b\right) = \frac{3}{2} R\left(p_c V_c - p_b V_b\right)$$
$$= \frac{3}{2}\left(2 p_0 \times 2V_0 - 2 p_0 V_0\right)$$
$$= 3 p_0 V_0.$$

As,

$$\Delta E_{bc} = W_{bc} + Q_{bc},$$
$$Q_{bc} = 5 p_0 V_0.$$

Therefore, the amount of heat added per cycle during the expansion stroke abc will be

$$Q = Q_{ab} + Q_{bc} = \frac{3}{2} p_0 V_0 + 5 p_0 V_0 = \frac{13}{2} p_0 V_0$$

$$= \frac{13}{2} \times 1.01 \times 10^5 \times 2.25 \times 10^{-2} \text{ J}$$

$$= 14.77 \text{ kJ.}$$

Let W_{da} be the work done on the gas during compression from volume V_1 to V_0 at pressure p_0.

$$W_{da} = -p_0 \left(V_0 - V_1 \right) = p_0 V_0.$$

Therefore, the total work done on the gas in each cycle will be

$$W = W_{bc} + W_{da} = -2 p_0 V_0 + p_0 V_0 = -p_0 V_0.$$

And therefore the total work done by the gas engine on the external system per cycle will be

$$-W = \left| W \right| = p_0 V_0 = 1.01 \times 10^5 \times 2.25 \times 10^{-2} \text{ J} = 2.27 \text{ kJ.}$$

(c)

The engine efficiency will be

$$e = \frac{\left| W \right|}{Q} = \frac{p_0 V_0}{\frac{13}{2} p_0 V_0} = \frac{2}{13} = 0.153.$$

That is the efficiency of the engine is 15.3%.

(d)

The highest temperature during the cycle will be at c,

$$T_H = \frac{p_c V_c}{R} = \frac{4 p_0 V_0}{R}.$$

The lowest temperature during the cycle will be at *a*,

$$T_L = \frac{p_0 V_0}{R}.$$

Therefore, the Carnot efficiency of an engine operating between the highest and the lowest temperatures during the cycle will be

$$e_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{p_0 V_0}{4 p_0 V_0} = \frac{3}{4} = 0.75.$$

That is 75%.