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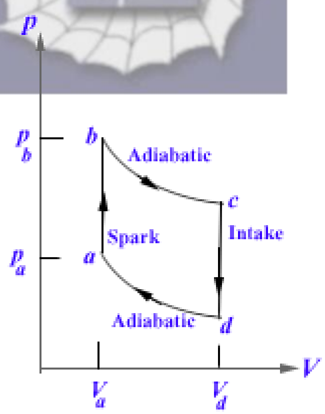
Problem 26.6 (RHK)

A gasoline internal combustion engine can be approximated by the cycle shown in the figure. We will assume that an ideal diatomic gas is used in the cycle and a compression ratio of 4:1 $(V_d = 4V_a)$. *We will further assume that* $p_b = 3p_a$. (a) *We have to determine the pressure and temperature of each of the vertex points of the pV diagram in terms of* p_a *and* T_a. (b) *We have to calculate the efficiency of the cycle.*



Solution:

(a)

It is given that the gasoline internal combustion engine can be approximated by an ideal diatomic gas. Therefore, the ratio of specific heats for this gas will be

$\gamma = \frac{7}{5} = 1.4$.(diatomic ideal gas)

Let the volume, pressure, and temperature of the gas at the point *a* in the *pV* diagram be V_a, p_a, and T_a, respectively.

It is given that at the point *b* in the cycle, $p_b = 3p_a$. From the pV diagram, we note that $V_b = V_a$.

From the ideal gas equation we note that temperature at *b* will be $T_b = 3T_a$.

Part *bc* of the cycle is an adiabatic process. In an adiabatic process

$pV^{\gamma} = \text{constant}$.

Therefore, we have

$p_c V_c^{\gamma} = p_b V_b^{\gamma}$.

Or

$$p_c = p_b \times \left(\frac{V_b}{V_c} \right)^{\gamma} = 3p_a \times \left(\frac{1}{4} \right)^{1.4} = 0.43 p_a,$$

as $V_c = 4V_b$.

Temperature at *c* will be

$T_c = 4 \times 0.43 T_a = 1.72 T_a$.

Also, the part *dc* of the cycle is an adiabatic process. Therefore,

$$p_d = p_a \times \left(\frac{V_a}{V_d}\right)^{\gamma} = p_a \times \left(\frac{1}{4}\right)^{1.4} = 0.14\, p_a.$$

And temperature at *d* will be

$$T_d = 0.14 \times 4T_a = 0.56T_a.$$

(b)

Process *ab* (spark) takes place at constant volume, therefore no work is done on the gas and the heat absorbed Q_1 will be equal to the increase in the internal energy. That is

$$Q_1 = \Delta E_{ab} = \frac{5}{2}nR\left(T_b - T_a\right) = \frac{5}{2}nR\left(3T_a - T_a\right) = 5nR.$$

We have assumed that the amount of gas undergoing the cyclic process is *n* moles and that it is an ideal diatomic gas.

Process *bc* is adiabatic. Therefore, there is no exchange of heat with the external system. In an adiabatic process the work done is given by

$$W = \frac{1}{\gamma - 1}\left(p_f V_f - p_i V_i\right).$$

Therefore,

$$W_{bc} = \frac{1}{\gamma - 1}\left(p_c V_c - p_b V_b \right) = \frac{1}{0.4}\left(0.43 p_a \times 4V_a - 3p_a \times V_a\right)$$
$$= -3.2\, p_a V_a.$$

And the work done on the gas during the adiabatic process *da* will be

$$W_{da} = \frac{1}{\gamma - 1}\left(p_a V_a - p_d V_d \right) = \frac{1}{0.4}\left(p_a V_a - 0.14 p_a \times 4V_a \right)$$
$$= 1.1\, p_a V_a.$$

For the sake of completeness, we calculate Q_2 the amount of heat released by the gas to the external system during the process *cd* (intake). We have

$$Q_2 = -\Delta E_{cd} = -\frac{5}{2} nR\left(T_d - T_c\right) = -\frac{5}{2} nR\left(0.56T_a - 1.72T_a\right)$$
$$= 2.9\, nRT_a.$$

The total amount of work done on the gas during one cycle will be

$$W = W_{bc} + W_{da} = -3.2\, p_a V_a + 1.1 p_a V_a = -2.1 p_a V_a$$
$$= -2.1 nRT_a.$$

As this work is negative, we can say that the work done is done by the engine during one cycle on the external system.

Therefore, the efficiency of the engine is

$$e = \frac{|W|}{Q_1} = \frac{2.1nRT_a}{5nRT_a} = 0.42.$$

That is the efficiency of the engine is 42%.