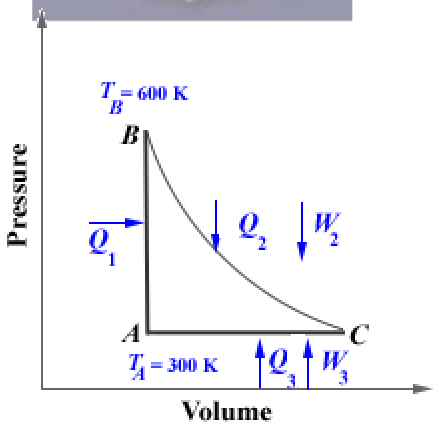
246.

<u>Problem 26.5 (RHK)</u>

One mole of a monatomic ideal gas initially at a volume of 10 L *and temperature* 300 K *is heated at constant volume to a temperature of* 600 K, *allowed to expand isothermally to its initial pressure, and finally compressed isobarically* (*that is, at constant pressure*) *to its original volume, pressure and temperature.* (a) *We have to compute the heat input to the system during one cycle;* (b) *find out the net work done by the gas during one cycle;* (c) *and find out the efficiency of this cycle.*



<u>Solution:</u>

We have drawn the cycle on a p-V diagram as shown in the figure.

Starting point of the cycle is at *A* in the diagram. We determine the pressure at *A* using the ideal gas equation of state.

$V_A = 10 \text{ L} = 10 \times 10^{-3} \text{ m}^3 = 10^{-2} \text{ m}^3.$

$T_A = 300 \text{ K}.$

$n = 1 \text{ mol}.$

Therefore, $p_A = \dfrac{nRT_A}{V_A} = \dfrac{8.31 \times 300}{10^{-2}} \text{ Pa} = 24.93 \times 10^4 \text{ Pa}.$

During the *AB* part of the cycle no work is done on the gas, as the gas is heated at constant volume. The quantity of heat absorbed by the gas, Q_1, will be equal to the change in the internal energy of the gas. As the ideal gas is monatomic,

$$Q_1 = \Delta E_{AB} = \frac{3}{2} R\left(T_B - T_A\right) = \frac{3 \times 8.31 \times \left(600 - 300\right)}{2} \text{ J}$$

$$= 3739.5 \text{ J}.$$

We next calculate Q_2 and W_2 during the isothermal expansion of the gas from *B* to *A*.

$$W_2 = -\int_{V_B}^{V_C} p dV = -\int_{V_B}^{V_C} \frac{RT_B}{V} dV = -RT_B \ln\left(\frac{V_C}{V_B}\right).$$

As the pressure at *C* is the same as that at *A* but the temperature of the gas at *C* is two times that at *A*, the volume of the gas at C will be two times of its volume at A. That is

$$V_C = 2V_A = 2V_B.$$

And

$$W_2 = -8.31 \times 600 \times \ln 2 \text{ J} = -3456.0 \text{ J}.$$

As the process BC is isothermal, there is no change in the internal energy of the gas. That is

$$\Delta E_{BC} = 0,$$

and from the first law of thermodynamics, we get

$$Q_2 = -W_2 = 3456.0 \text{ J}.$$

The CA part of the cycle takes place isobarically, that is at constant pressure. Therefore, the work done on the gas during this part of the cycle

$$W_3 = -p_A\left(V_A - V_C\right) = -24.93 \times 10^4 \times \left(-10^2\right) \text{ J}$$
$$= 2493 \text{ J}.$$

And the change in internal energy of the gas during CA will be

$$\Delta E_{CA} = \frac{3}{2} R\left(T_A - T_C\right) = \frac{3 \times 8.31 \times (300 - 600)}{2} \text{ J}$$
$$= -3739.5 \text{ J}.$$

From the first law of thermodynamics, we have

$$\Delta E_{CA} = Q_3 + W_3.$$

Therefore, the heat absorbed by the gas during *CA* will be

$$Q_3 = \Delta E_{CA} - W_3 = (-3739.5 - 2493)\ \text{J}$$
$$= -6232.5\ \text{J}.$$

As Q_3 is negative, heat is released by the gas into the system during this part of the cycle.

(a)

The total heat absorbed by the gas in one cycle will be

$$Q_{in} = Q_1 + Q_2 = (3739.5 + 3456.0)\ \text{J} = 719.5\ \text{J}.$$

(b)

The net work done by the gas on the external system during one cycle will be

$$W = -W_2 - W_3 = (3456.0 - 2493)\ \text{J} = 963\ \text{J}.$$

(c)

Efficiency of the cycle will be

$$e = \frac{W}{Q_{in}} = \frac{963}{7195.5} = 0.134.$$

That is 13.4%.