243.

Problem 25.59 (RHK)

Ice has formed on a shallow pond and a steady state has been reached with the air above the ice at -5.20^0C *and the bottom of the pond at* 3.98^0C. *The total depth of ice* $+$*water is* 1.42 m. *We have to find how thick the ice is. We may assume that the thermal conductivities of ice and water are* 1.67 *and* 0.502 W m^{-1} K^{-1}.

Solution:

Let the thickness of ice be *h* m. The total depth of ice $+$water is 1.42 m. Therefore, the water depth below the ice surface to the bottom of the tank will be $(1.42\text{ m} - h)$.

Temperature of water in contact with the ice slab will be 0^0C. Temperature of air above the ice surface is -5.20^0C. Therefore the temperature difference across the ice-surface is $\Delta T_{ice} = 5.2$ K.

Temperature at the bottom of the pond is 3.98^0C. Therefore, the temperature difference between that of the

bottom of the pond and the bottom layer of the ice surface will be $\Delta T_{water} = 3.98$ K.

Thermal conductivity of water is

$k_{water} = 0.502$ W m^{-1} K^{-1}.

The thermal conductivity of ice is

$k_{ice} = 1.67$ W m^{-1} K^{-1}.

In the condition of steady state the rate of loss of heat from the bottom of the pond per meter^2 will be equal to the rate of heat flow per meter^2 through the ice to the air above the ice surface. That is

$$k_{water} \times \frac{\Delta T_{water}}{1.42 \text{ m} - h} = k_{ice} \times \frac{\Delta T_{ice}}{h},$$

or

$$0.502 \times \frac{3.98}{1.42 \text{ m} - h} = 1.67 \times \frac{5.2}{h},$$

or

$$1.998h = 12.33 \text{ m} - 8.684h,$$

or

$$h = 1.15 \text{ m}.$$