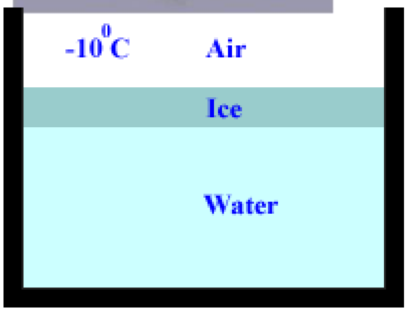
242.

Problem 25.58 (RHK)

A container of water has been outdoors in cold weather until a 5.0-cm-thick *slab of ice has been formed on its surface. The air above the ice is at* -10^0C. *We have to calculate the rate of formation of ice (in centimetres per hour) on the bottom surface of the ice slab. For the thermal conductivity and density of ice we may use the values* $1.7\ \text{W m}^{-1}\ \text{K}^{-1}$ *and* $0.92\ \text{g cm}^{-3}$, *respectively. We may assume that no heat flows through the walls of the tank.*



Solution:

As the temperature of water inside the tank will be 0^0C and the outside temperature is -10^0C, the temperature difference across the two faces of the ice slab will be,

$\Delta T = 10\ \mathrm{C}^0$. The width of the ice slab is 5 cm. The rate of flow of heat from water in the tank through each unit cross-section of the ice slab can be calculated from the heat conductivity equation

$$H = -kA\frac{\Delta T}{\Delta x} \ .$$

Data relevant to the problem are

$k = 1.7\ \mathrm{W\,m^{-1}\ K^{-1}}$,

$A = 1.0\ \mathrm{m}^2$,

$\Delta T = 10\ \mathrm{K}$,

$\Delta x = 5.0\times 10^{-2}\ \mathrm{m}$.

Therefore, rate of heat flow out through the ice slab will be

$$H = \frac{1.7\times 10}{5.0\times 10^{-2}}\ \mathrm{W\,m^{-2}}$$

$$= 0.34\times 10^3 \times 3600\ \mathrm{J\ m^{-2}\ hour^{-1}}$$

$$= 12.24\times 10^5\ \mathrm{J\ m^{-2}\ hour^{-1}}.$$

The rate of freezing can be calculated by noting that the latent heat of fusion of water is

$L_{fusion-water} = 333\times 10^3\ \mathrm{J\ kg^{-1}}$.

Density of water at $0^0\mathrm{C}$ is $\rho = 0.92\times 10^3\ \mathrm{kg\ m^{-3}}$.

Let Δh m be the ice thickness that is formed in one hour. This implies that amount of heat that is flowing out of the tank in one hour is

$$Q = 0.92\times10^{3}\times333\times10^{3}\times\Delta h \text{ J}=306\times10^{6}\times\Delta h \text{ J}.$$

And we know the amount of heat that is flowing out through 1 m^2 of the ice slab per hour is equal to *H*. We, therefore, have the relation

$$306\times10^{6}\times\Delta h \text{ J} = 12.24\times10^{5} \text{ J},$$

and

$$\Delta h = 4.0\times10^{-3} \text{ m} = 0.4 \text{ cm}.$$

Therefore, the rate of formation of ice at the bottom of the ice slab is 0.4 cm per hour.