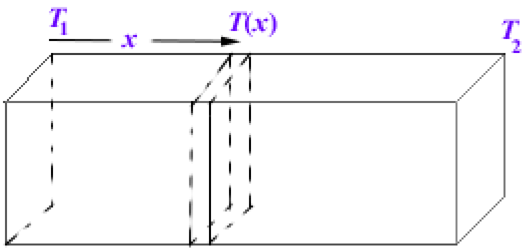
241.

<u>Problem 25.63 (RHK)</u>

At low temperatures (below about 50 K)*, the thermal conductivity of a metal is proportional to the absolute temperature; that is* $k = aT$*, where a is a constant with a numerical value that depends on the particular metal. We have to show that the rate of heat flow through the rod of length L and cross-sectional area A whose ends are at temperatures* T_1 *and* T_2 *is given by*

$$H = \frac{aA}{2L}\left(T_1^2 - T_2^2\right).$$

We may ignore the heat loss from the surface.



<u>Solution:</u>

Let the variation of temperature along the length of the rod be described by the function $T(x)$, where x is the length measured from the end which is kept at temperature T_1.

Under the assumption that there is no heat loss from the surface of the rod, heat flow *H* will be independent of *x*. Thermal conductivity varies with temperature and is given by the function

$k = aT$,

where *a* has a constant value that depends on the metal through which heat is being conducted. From the law of thermal conduction, we have

$$H = -aAT(x)\frac{dT(x)}{dx} \ ,$$

where *A* is the cross-sectional area of the rod. As *H* has a constant value along the length of the rod, that is it does not depend on *x*, we have

$$T(x)\frac{dT(x)}{dx} = c \ ,$$

or

$$\frac{dT^2(x)}{dx} = 2C.$$

On integrating this linear differential equation, we get

$$T^2(x) = 2cx + d.$$

We determine the constants c and d by using the boundary conditions

$$T\left(x=0\right)=T_1\ ,$$

and

$$T\left(x=L\right)=T_2.$$

We find

$$d=T_1^2,$$

and

$$c=\frac{T_2^2-T_1^2}{2L}.$$

Therefore,

$$H=\frac{aA}{2L}\left(T_1^2-T_2^2\right).$$