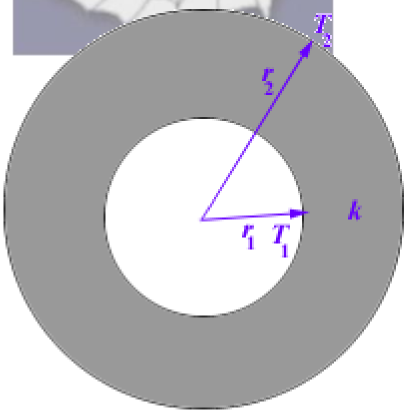
240.

Problem 25.61 (RHK)

Assuming k is constant, we have to show that the radial rate of flow of heat in a substance between two concentric spheres is given by

$$H = \frac{(T_1 - T_2)4\pi k r_1 r_2}{r_2 - r_1},$$

where the inner sphere has a radius r_1 *and temperature* T_1*, and the outer surface has a radius* r_2 *and temperature* T_2.



Solution:

The surface of the inner sphere of radius r_1 is maintained at temperature T_1. The surface of the outer sphere of radius r_2 is maintained at temperature T_2. When thermal equilibrium is established let us assume that radial temperature gradient is described by the function $\frac{dT(r)}{dr}$.

Let us consider a concentric spherical surface of radius r from the centre of the spheres. The rate of heat flow across this surface will be given by the equation

$$H = -k \times \left(4\pi r^2\right) \times \frac{dT}{dr},$$

or

$$\frac{Hdr}{4\pi k r^2} = -dT.$$

Integrating this equation between the limits r_1, T_1, and r_2, T_2, we get

$$\int_{r_1}^{r_2} \frac{Hdr}{4\pi k r^2} = -T_2 + T_1,$$

or

$$\frac{H}{4\pi k} \times \left(-\frac{1}{r}\right)\Bigg]_{r_1}^{r_2} = T_1 - T_2 \ .$$

Or

$$\frac{H}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=T_1-T_2$$

or

$$H=\frac{4\pi k\, r_1 r_2\left(T_1-T_2\right)}{\left(r_2-r_1\right)}.$$