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<u>Problem 25.53 (RHK)</u>

A long tungsten heater wire is rated at 3.08 kW m^{-1} *and is* 0.520 mm *in diameter. It is embedded across the axis of a ceramic cylinder of diameter* 12.4 cm. *When operated at the rated power, the wire is at* 1480^0C, *the outside of the cylinder is at* 22.0^0C. *We have to calculate the thermal conductivity of the ceramic.*

<u>Solution:</u>

We will workout the heat flow in the radial direction across the surface of a ceramic cylinder of length, $L = 1.0$ m, and of diameter $d_2 = 0.124$ m. It is given that the diameter of the tungsten wire fixed lengthwise at the axis of the ceramic cylinder is $d_1 = 0.520 \times 10^{-3}$ m.

The tungsten wire is being heated at its rated power, $H = 3.08 \times 10^3 \text{W}$.

Temperature of the wire is $T_1 = 1480^0\text{C}$.

Temperature of the surface of the ceramic cylinder is $T_2 = 22^0\text{C}$.

At thermal equilibrium a radial temperature gradient will be established. Let us call it $\frac{dT(r)}{dr}$.

At thermal equilibrium the rate of flow of heat across a cylindrical surface of length 1 m at radial distance r from the axis will be $H = 3.08\times10^{3}\,\mathrm{W}$. The area of the cylindrical surface of length 1 m across which the heat is flowing outward in the radial direction is $2\pi r\ \mathrm{m}^2$.

Let the thermal conductivity of the ceramic be *k*.

According to the law of thermal conduction

$$H = -k\times(2\pi r)\frac{dT}{dr},$$

or

$$\frac{H}{2\pi k}\times\frac{dr}{r} = -dT.$$

Integrating the above expression, we get

$$\frac{H}{2\pi k}\times\ln\left(\frac{r_2}{r_1}\right) = -(T_2 - T_1).$$ And

$$k = \frac{H}{2\pi(T_1 - T_2)}\times\ln\left(\frac{r_2}{r_1}\right)$$

$$= \frac{3.08\times10^{3}}{2\pi\times1458}\ln\left(\frac{12.4}{5.2\times10^{-2}}\right)\ \mathrm{W\ m^{-1}\ K^{-1}}$$

$$= 1.84\ \mathrm{W\ m^{-1}\ K^{-1}}.$$