237.

Problem 25.47 (RHK)

The average rate at which heat flows out through the surface of the Earth in North America is 54 mW m^{-2}, *and the average thermal conductivity of the near surface rocks is* $2.5 \text{ W m}^{-1} \text{ K}^{-1}$. *Assuming a surface temperature of* 10^0 C, *we have to estimate the temperature at a depth of* 33 km (*near the base of the crust*). *We may ignore the heat generated by radioactive elements and can also ignore the curvature of the Earth.*

Solution:

This is a problem of heat conduction. The rate of flow of heat across a slab of width *L*, cross-sectional area *A*, when its two ends are maintained at temperatures T_H and T_L, is given by

$$H = kA\frac{\left(T_H - T_L\right)}{L}.$$

Data of the problem are

average thermal conductivity of the rocks

$k = 2.5 \text{ W m}^{-1} \text{ K}^{-1}$, rate of flow of heat across one square meter surface of the rocks is $H = 54\times10^{-3}$ W,

$A = 1\ \text{m}^2$,

length across which the heat is conducted through the rocks to the surface of the Earth is $L = 33\times10^{3}$ m,

temperature of the surface of the Earth $T_L = 10^0\text{C}$,

and let the temperature at the depth of 33 km be $T_H\,^0\text{C}$.

We therefore have

$$T_H - T_L = \frac{54\times10^{-3}\times33\times10^{3}}{2.5}\ ^0\text{C}$$

$$= 712.8^0\text{C},$$

or

$$T_H = 722.8^0\text{C}.$$