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Problem 25.45 (RHK)

In a motor cycle engine, after combustion occurs in the top of the cylinder, the piston is forced down as the mixture of gaseous products undergoes an adiabatic expansion. We have to find the average power involved in this expansion when the engine is running at 4000 rpm, *assuming that the gauge pressure immediately after combustion is* 15.0 atm, *the initial volume is* 50.0 cm^3, *and the volume of the mixture at the bottom of the stroke is* 250 cm^3. *We may assume that the gases are diatomic and that the time involved in the expansion is one-half that of the total cycle.*

Solution:

The problem is around the concept of work done during an adiabatic expansion.

The data of the problem are

Initial pressure of the gas,

$$p_i = 15.0 \text{ atm} = 15 \times 1.013 \times 10^5 \text{ Pa} = 15.195 \times 10^5 \text{ Pa}.$$

Initial volume of the gas, $V_i = 50.0\ \text{cm}^3 = 50.0 \times 10^{-6}\ \text{m}^3$.

Final volume of the gas that is after the adiabatic expansion, $V_f = 250\ \text{cm}^3 = 250 \times 10^{-6}\ \text{m}^3$.

As the gases involved in the process are composed of diatomic molecules, the ratio of heat capacities $\gamma = 7/5$.

As the expansion of the gases occurs adiabatically, the pressure after expansion can be determined from the relation

$$p_f = p_i \times \left(\frac{V_i}{V_f}\right)^{\gamma} = 15.195 \times 10^5 \times \left(\frac{50}{250}\right)^{7/5}\ \text{Pa}$$

$$= 1.596 \times 10^5\ \text{Pa}.$$

The work done on a gas in an adiabatic process is given by the relation

$$W = \frac{p_f V_f - p_i V_i}{\gamma - 1}$$

$$= \frac{\left(1.596 \times 10^5 \times 250 \times 10^{-6} - 15.195 \times 10^5 \times 50 \times 10^{-6}\right)}{\frac{7}{5} - 1}\ \text{J}$$

$$= -90.18\ \text{J}.$$

As the work done on the gas is negative, work is done by the gas on the external system.

We will next use the information that the time involved in the adiabatic expansion of the gases on combustion is one-half that of the cycle. The engine of the motor cycle is running at 4000 rpm, therefore, the duration of one-half of the cycle is

$$t_{1/2} = \frac{60}{2 \times 4000}\ \text{s} = 7.5 \times 10^{-3}\ \text{s}.$$

Therefore, the average power involved in the motor cycle is

$$P = \frac{|W|}{t_{1/2}} = \frac{90.18}{7.5 \times 10^{-3}}\ \text{J s}^{-1}$$

$$= 12.0\ \text{kW}.$$