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<u>Problem 25.37 (RHK)</u>

A quantity of ideal monatomic gas consists of n moles initially at temperature T_1. *The pressure and volume are then slowly doubled in such a manner as to trace out a straight line on pV diagram. In terms of n, R, and* T_1, *we have to find* (a) *W*, (b) ΔE_{int}, *and* (c) *Q*. (d) *If one were to define an equivalent specific heat for the process, we have to calculate its value.*

<u>Solution:</u>

As the change of pressure and volume is linear in the *pV* diagram and the pressure doubles when the volume doubles, the equation of the straight line relating the pressure-volume is

$$p = \left(\frac{p_1}{V_1}\right) \times V.$$

Also, the initial temperature of the gas is T_1 and the quantity of the gas is *n* moles, we have

$$p_1 V_1 = nRT_1.$$

(a)

The work done on the gas during the slow change described by the linear relation will be

$$W = -\int_{V_1}^{2V_1} p dV = -\int_{V_1}^{2V_1} \frac{p_1}{V_1} \times V dV = -\frac{3}{2} p_1 V_1 = -\frac{3}{2} nRT_1.$$

(b)

The final temperature of the gas T_f can be obtained from the equation of state.

$$nRT_f = p_f V_f = 4 p_1 V_1 = 4nRT_1,$$

and

$$T_f = 4T_1.$$

The change in the internal energy of a monatomic ideal gas can be obtained from the initial and final temperatures.

$$\Delta E_{\text{int}} = E_{\text{int}\, f} - E_{\text{int}\, i} = \frac{3}{2} nR\left(T_f - T_i\right) = \frac{3}{2} nR\left(4T_1 - T_1\right) = 4.5 nRT_1.$$

(c)

The heat absorbed Q can be calculated from the first law of thermodynamics,

$$\Delta E_{\text{int}} = W + Q,$$

and

$$Q = \Delta E_{\text{int}} - W = \frac{9}{2} nRT_1 - \left(-\frac{3}{2} nRT_1 \right) = 6nRT_1.$$

(d)

We define the equivalent specific heat

$$C = \frac{Q}{n\Delta T} = \frac{6nRT_1}{n \times 3T_1} = 2R.$$