229.

<u>Problem 25.19 (RHK)</u>

A 21.6-g *copper ring has a diameter of* 2.54000 cm *at its temperature of* 0^0C. *An aluminium sphere has a diameter of 2.54533 cm at its temperature of* 100^0C. *The sphere is placed on top of the ring, and the two are allowed to come to thermal equilibrium, no heat being lost to the surroundings. The sphere just passes through the ring at the equilibrium temperature. We have to find the mass of the sphere.*

<u>Solution:</u>

In solving this problem we will use two concepts: (i) thermal linear expansion, and (ii) conservation of heat energy.

It is given that the temperature of the copper ring is $0^{0}\mathrm{C}$ and its diameter at $0^{0}\mathrm{C}$ is

$$d_{ring}\left(0^{0}\mathrm{C}\right)=2.54000 \text{ cm}.$$

It is given that the temperature of the aluminium sphere is $100^{0}\mathrm{C}$ and its diameter is

$$d_{sphere}=2.54533 \text{ cm}.$$

Let the equilibrium temperature of the ring-sphere system after the two have been in thermal contact be $T^{0}\mathrm{C}$.

The linear thermal expansion coefficient of copper is

$$\alpha_{Cu}=17\times10^{-6} \text{ per } \mathrm{C}^{0},$$

and the linear thermal expansion coefficient of aluminium is

$$\alpha_{Al}=23\times10^{-6} \text{ per } \mathrm{C}^{0}.$$

Using the property of linear expansion we first determine the diameter of the ring and that of the aluminium sphere at the equilibrium temperature.

$$d_{rimg}\left(T^{0}\mathrm{C}\right)=2.5400\left(1+17\times10^{-6}T\right)$$

and

$$d_{sphere}=2.54533\left(1-23\times10^{-6}T\right).$$

As the sphere just passes through the ring at $T^0\text{C}$, we have the condition

$$d_{ring}\left(T^0\text{C}\right) = d_{sphere}\left(T^0\text{C}\right),$$

or

$$2.5400\left(1+17\times10^{-6}T\right) = 2.54533\left(1-23\times10^{-6}T\right).$$

It is a linear algebraic equation for *T*. We find

$$T = 34.1^0\text{C}.$$

For we determining the mass of the aluminium sphere we will use the conservation of heat energy.

The specific heat capacity of copper is

$$c_{Cu} = 387 \text{ J kg}^{-1} \text{ K}^{-1},$$

and that of aluminium is

$$c_{Al} = 900 \text{ J kg}^{-1} \text{ K}^{-1}.$$

The mass of the copper ring is 21.6×10^{-3} kg.

Let the mass of the aluminium sphere be m kg.

Change of heat energy of the aluminium sphere will be

$$\begin{aligned}\Delta Q_{sphere} &= 900\times(34.1-100)\times m \text{ J} \\ &= -59{,}310\times m \text{ J.}\end{aligned}$$

Change of heat energy of the copper ring will be

$$\begin{aligned}\Delta Q_{ring} &= 21.6\times10^{-3}\times387\times34.1 \text{ J} \\ &= 285.05 \text{ J.}\end{aligned}$$

From the conservation of energy, we have the condition

$$\Delta Q_{sphere} + \Delta Q_{ring} = 0,$$

or

$$m = \frac{285.05}{59,310} = 4.81\times 10^{-3} \text{ kg} = 4.81 \text{ g}.$$