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Problem 20.55 P (HRW)

We have to compute (a) *the temperature at which the rms speeds of molecular hydrogen and molecular oxygen to be equal to the speed of escape from the Earth's surface.* (b) *We have to repeat the calculations for the Moon, assuming the gravitational acceleration on its surface to be* 0.16 g. *If the temperature high in the Earth's upper atmosphere is about* 1000 K, *we have to answer whether we expect to find much hydrogen there and much oxygen there.*

Solution:

The escape speed from the surface of a spherical object of mass M and radius R is given by the requirement that the magnitude of the kinetic energy and the gravitational potential energy of a particle to escape to infinity from the surface of the object should be equal. That is

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R},$$

or

$$v_{esc} = \sqrt{\frac{2GM}{R}}.$$

For the Earth $M = 5.98 \times 10^{24}$ kg ,and $R = 6.37 \times 10^{6}$ km.

$$v_{esc}(\text{Earth}) = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^{6}}} \text{ m s}^{-1}$$

$$= 11.19 \text{ km s}^{-1}.$$

Mass of a hydrogen molecule is

$$m_{H_2} = \frac{2}{6.02 \times 10^{23}} \text{ g} = 0.332 \times 10^{-26} \text{ kg}.$$

(a)

Therefore, the temperature at which the rms speed of a hydrogen molecule will equal the escape speed from the surface of the Earth will be

$$T_{H_2} = \frac{m_{H_2} v_{esc}^{2}(\text{Earth})}{3k} = \frac{0.332 \times 10^{-26} \times (11.2 \times 10^{3})^{2}}{3 \times 1.38 \times 10^{-23}} \text{ K}$$

$$= 1.0 \times 10^{4} \text{ K}.$$

As mass of an oxygen molecule is 16 times that of a hydrogen molecule, we have

$$T_{O_2} = 16.0 \times 10^{4} \text{ K}.$$

Therefore, in the Earth's upper atmosphere where the temperature is about 1000 K, the fraction of the hydrogen molecules with speed greater than the escape from the

Earth will be more than the fraction of oxygen molecules with speed more than the escape speed from the Earth.

(b)

We repeat the above calculation for the Moon. Data of the moon are

radius of the moon $R_{moon} = 1.74 \times 10^6$ m,

acceleration due to gravity on the Moon's surface

$$\frac{GM_{moon}}{R_{moon}^2} = 0.16 \text{ g}.$$

Therefore, the speed of escape from the surface of the Moon

$$v_{esc}^2(\text{Moon}) = \frac{2GM_{moon}}{R_{moon}^2} \times R_{moon} = 2 \times 0.16 \times 9.81 \times 1.74 \times 10^6 \text{ m}^2 \text{ s}^{-2}$$

$$= 5.46 \times 10^6 \left(\text{m s}^{-1}\right)^2,$$

or

$$v_{esc}(\text{Moon}) = 2.34 \times 10^3 \text{ m s}^{-1}.$$

At the Moon the temperature of the hydrogen gas for which the rms speed will be equal to the escape speed from the surface of the Moon will be

$$T_{H_2} = \frac{0.332 \times 10^{-26} \times \left(2.34 \times 10^3\right)^2}{3 \times 1.38 \times 10^{-23}} \text{ K}$$

$$= 438 \text{ K}.$$

And,

$$T_{O_2} = 16 \times T_{H_2} = 16 \times 438 \text{ K} = 7008 \text{ K}.$$