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Problem 24.31 (RHK)

Very small particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. We may assume that the grains are uniform spheres of diameter 4.0×10^{-6} cm *and density* 1.0 g cm^{-3}, *and that the temperature of the gas is* 100 K. *we have to find* (a) *the root means square speed of the grains between collisions and* (b) *the approximate rate* (rev s^{-1}) *at which the grains are spinning.*

Solution:

Data of the problem are

diameter of each grain $d = 4.0\times10^{-6}$ cm,

density of grains is $\rho = 1.0$ g cm^{-3}.

Therefore, mass of a grain,

$$m = \frac{4\pi}{3} \times \left(\frac{d}{2}\right)^3 \times \rho = \frac{4\pi}{3} \times \left(2 \times 10^{-6}\right)^3 \times 1.0 \text{ g}$$

$$= 3.35 \times 10^{-17} \text{ g} = 3.35 \times 10^{-20} \text{ kg}.$$

As the grains are immersed in interstellar gas at temperature 100 K, they will undergo Brownian motion because of random collisions of the hydrogen molecules of the gas. The average translational kinetic energy of a grain at temperature T will become

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT.$$

And, the average rotational energy of each grain, assuming that it can be considered as a rigid sphere of diameter d, will be

$$\frac{1}{2} I \omega^2 = \frac{3}{2} kT,$$

where the rotational inertia I is

$$I = \frac{2}{5} m \left(\frac{d}{2}\right)^2,$$

and ω is the angular speed of rotation of a grain.

(a)

We calculate v_{rms} of the grains using

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times 1.38\times 10^{-23}\times 100}{3.35\times 10^{-20}}}\ \text{m s}^{-1}$$

$$= 0.35\ \text{m s}^{-1} = 35\ \text{cm s}^{-1}.$$

(b)

We calculate angular speed ω by using the relation

$$\frac{1}{2}I\omega^2 = \frac{3}{2}kT,\ \text{and}\ I = \frac{2}{5}m\left(\frac{d}{2}\right)^2.$$

We have

$$\omega = \sqrt{\frac{30kT}{md^2}} = \sqrt{\frac{30\times 1.38\times 10^{-23}\times 100}{3.35\times 10^{-20}\times\left(4\times 10^{-8}\right)^2}}\ \text{rad s}^{-1}$$

$$= 2.78\times 10^{8}\ \text{rad s}^{-1}$$

$$= \frac{2.78\times 10^{8}}{2\pi}\ \text{rev s}^{-1} = 4.4\times 10^{6}\ \text{rev s}^{-1}.$$