221.

<u>Problem 24.9 (RHK)</u>

We have to estimate the frequency for which the wavelength of sound will be of the order of the mean free path in nitrogen at 1.02 atm *pressure and* $18.0\,^{0}\mathrm{C}$ *temperature. We can take the diameter of the nitrogen molecule to be* 315 pm.

<u>Solution:</u>

Data of the problem are

$p = 1.02 \text{ atm} = 1.02 \times 1.01 \times 10^{5} \text{ Pa} = 1.03 \times 10^{5} \text{ Pa},$

$T = 18.0\,^{0}\mathrm{C} = 291.16,$

and

$d = 315 \text{ pm} = 315 \times 10^{-12} \text{ m}.$

Number density ρ_n is given by

$$\rho_n = \frac{p}{kT} = \frac{1.03 \times 10^{5}}{1.38 \times 10^{-23} \times 291.16} \text{ molecules. m}^{-3}$$
$$= 2.56 \times 10^{25} \text{ molecules. m}^{-3}.$$

Mean free path will be

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 \rho_n} = \frac{1}{\sqrt{2}\pi \times \left(315\times10^{-12}\right)^2 \times 2.564\times10^{25}} \text{ m}$$

$$= 8.85\times10^{-8} \text{ m.}$$

Velocity of sound is given in terms of thermodynamic quantities as

$$V = \sqrt{\frac{\gamma RT}{M}}.$$

For nitrogen gas made up of N_2 molecules, the molar mass is

$M = 2\times14 \text{ g} = 28\times10^{-3} \text{ kg},$

and as the nitrogen gas is diatomic

$\gamma = 1.4$.

Therefore, the velocity of sound in the nitrogen gas at 291.16 K will be

$$V = \sqrt{\frac{1.4\times8.31\times291.16}{28\times10^{-3}}} \text{ m s}^{-1} = 347.8 \text{ m s}^{-1}.$$

Therefore, the frequency of sound for which the wavelength will be of the order of the mean free path will be

$$\nu = \frac{347.8}{8.85\times10^{-8}} = 3.39\times10^{9} \text{ Hz} = 3.93 \text{ GHz.}$$