218.

Problem 23.51 (RHK)

An ideal gas experiences an adiabatic compression from $p = 122\ \mathrm{kPa}$, $V = 10.7\ \mathrm{m}^3$, $T = -23^0\mathrm{C}$ *to* $p = 1450\ \mathrm{kPa}$, $V = 1.36\ \mathrm{m}^3$. *We have to calculate* (a) *the value of* γ*;* (b) *the final temperature;* (c) *quantity of gas present in moles;* (d) *the total translational kinetic energy per mole before and after the compression; and* (e) *the ratio of the rms speed before to that after the compression.*

Solution:

Data of the problem are

$p_i = 122\ \mathrm{kPa}$,

$V_i = 10.7\ \mathrm{m}^3$,

$T_i = -23^0\mathrm{C} = 250.16\ \mathrm{K}$,

$p_f = 1450\ \mathrm{kPa}$,

and

$V_f = 1.36\ \mathrm{m}^3$.

(a)

In an adiabatic process pressure and volume vary as per the relation

$$p_i V_i^{\gamma} = p_f V_f^{\gamma},$$

or

$$\gamma = \frac{\ln\left(p_f / p_i\right)}{\ln\left(V_i / V_f\right)} = \frac{\ln\left(1450/122\right)}{\ln\left(10.7/1.36\right)} = 1.20.$$

(b)

In an adiabatic process temperature and volume vary as per the relation

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1},$$

or

$$T_f = T_i \times \left(\frac{V_i}{V_f}\right)^{\gamma-1} = 250.16 \times \left(\frac{10.7}{1.36}\right)^{0.2} = 377.9 \text{ K} = 1.05^{0}\text{C}.$$

(c)

Amount of gas in moles, μ, can be calculated from the ideal gas equation of state

$$pV = \mu RT,$$

or

$$\mu = \frac{pV}{RT} = \frac{122\times10^{3}\times10.7}{8.3145\times250.16}\ \text{mol} = 628\ \text{mol}.$$

(d)

Total internal energy, which for monatomic molecules is equal to the total translational kinetic energy, is given by

$$U = \frac{3}{2}\mu RT.$$

Therefore, the initial total kinetic energy of the molecules of the gas is

$$K_i = \frac{3}{2}\times628\times8.3145\times250.16\ \text{J} = 1.96\times10^{6}\ \text{J}.$$

The final total kinetic energy of the molecules of the gas will be

$$K_f = \frac{3}{2}\times628\times8.3145\times378\ \text{J} = 2.96\times10^{6}\ \text{J}.$$

(e)

Ratio of the rms initial and final speeds of the molecules can be calculated from the average kinetic energy of molecules at temperature T, which is given by the relation

$$\frac{1}{2}mv^2 = \frac{3}{2}kT,$$

where *m* is the mass of molecules of the gas. We therefore have

$$\frac{v_i^2}{v_f^2} = \frac{T_i}{T_f},$$

or

$$\frac{v_i}{v_f} = \left(\frac{T_i}{T_f}\right)^{1/2} = \left(\frac{250.16}{378}\right)^{1/2} = 0.813.$$