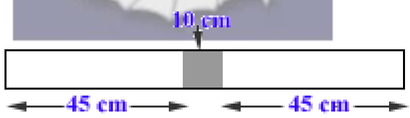
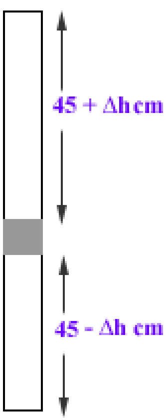
213.

Problem 23. 47 (RHK)

A thin tube, sealed at both ends is 1.00 m *long. It lies horizontally, the middle* 10.0 cm *containing mercury and the two equal ends containing air at standard atmospheric pressure. The tube is turned to a vertical position. We have to find the length by which the mercury will be displaced. We can assume that* (a) *the process is isothermal and* (b) *adiabatic. (For air* $\gamma = 1.40$)*. We have to answer which assumption is more reasonable.*



Solution:



It is given that a 1.0 m tube sealed at both ends contains a pallet of mercury 10 cm long located at the mid-point of the tube. The two equal ends contain air at standard atmospheric pressure.

The tube is turned to a vertical position. Position of the mercury pallet will shift down by an amount Δh cm and equilibrium will be achieved because of unequal air pressures in the upper and the lower parts of the tube. Let the changed length of the upper part of the tube be $(45+\Delta h)$ cm and that of the lower part be $(45-\Delta h)$ cm.

Let us assume that the thermodynamic change is adiabatic. For an adiabatic process

$$PV^{\gamma} = \text{const.}$$

Pressure in the upper part of the tube will become

$$P_u = \left(\frac{45}{45+\Delta h}\right)^{\gamma} \times 1.01\times10^5 \text{ Pa} \approx \left(1-\frac{\gamma\Delta h}{45}\right)\times 1.01\times10^5 \text{ Pa}.$$

And the pressure in the lower part of the tube will become

$$P_l = \left(\frac{45}{45-\Delta h}\right)^{\gamma} \times 1.01\times10^5 \text{ Pa} \approx \left(1+\frac{\gamma\Delta h}{45}\right)\times 1.01\times10^5 \text{ Pa}.$$

We require that for equilibrium pressure in the upper part of the tube plus the pressure due to the mercury pallet should be equal to the pressure in the lower part of the tube. This condition gives the relation

$$\frac{2\gamma\Delta h}{45} = \frac{13.6\times10^3\times9.8\times0.1}{1.01\times10^5},$$

or

$$\Delta h = \frac{2.97}{\gamma} \text{ cm.}$$

For air $\gamma = 1.40$. Therefore, if the process is adiabatic the mercury pallet will slip down by

$$\Delta h_{adiabaic} = \frac{2.97}{1.4} \text{ cm} = 2.12 \text{ cm.}$$

If the process is isothermal, we can obtain the change in length by using $\gamma = 1$, and the mercury pallet will slip down by 2.97 cm.

When the tube is turned vertically the exchange of heat with the surroundings will be negligible and considering that the thermodynamic process occurs adiabatically is a better assumption.