208.

Problem 23. 21 (RHK)

At $44.0\,^{0}\mathrm{C}$ *and* 1.23×10^{-2} atm *the density of a gas is* 1.32×10^{-5} g cm^{-3}. (a) *We have to find* v_{rms} *for the gas molecules.* (b) *We have to find the molar mass of the gas and identify it.*

Solution:

For a gas at temperature T the average energy of the molecules is

$$mv_{rms}^{2}/2 = 3kT/2,$$

or

$$v_{rms}^{2} = 3kT/m,$$

where k is the Boltzmann constant and m is the mass of a molecule. Ideal gas equation is

$$PV = NkT,$$

where N is the total number of molecules occupying the volume V at pressure P and at temperature T. Therefore,

$$v_{rms}^{2} = \frac{3PV}{MN} = \frac{3P}{\rho},$$

where M is the mass of the gas and ρ is its density.

The pressure of the gas is $P = 1.23\times10^{-2}$ atm , its density is $\rho = 1.32\times10^{-5}$ g cm^{-3}. Therefore,

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\times1.23\times10^{-2}\times1.01\times10^{5}}{1.32\times10^{-5}\times10^{3}}} \text{ m s}^{-1}$$

$$= 5.31\times10^{2} \text{ m s}^{-1}.$$

One mole of gas has N_A molecules. The total kinetic energy of one mole of gas will therefore be

$$mN_A v_{rms}^2/2 = 3kN_A T/2 = 3RT/2,$$

and

$$mN_A = \frac{3RT}{v_{rms}^2}.$$

Mass of one mole of the gas will therefore be

$$mN_A = \frac{3RT}{v_{rms}^2} = \frac{3\times8.3145\times(273.16+44)}{(5.31\times10^{2})^{2}} \text{ kg} = 28.0 \text{ g}.$$

Therefore, the gas is nitrogen (N_2).