

208.

Problem 23. 21 (RHK)

At 44.0⁰C and 1.23×10⁻² atm the density of a gas is 1.32×10⁻⁵ g cm⁻³. (a) We have to find v_{rms} for the gas molecules. (b) We have to find the molar mass of the gas and identify it.

Solution:

For a gas at temperature T the average energy of the molecules is

$$mv_{rms}^2/2 = 3kT/2,$$

or

$$v_{rms}^2 = 3kT/m,$$

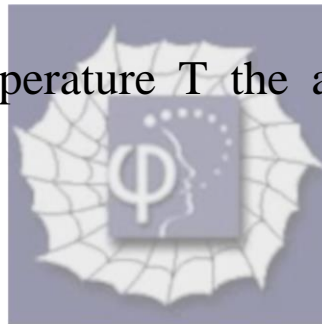
where k is the Boltzmann constant and m is the mass of a molecule. Ideal gas equation is

$$PV = NkT,$$

where N is the total number of molecules occupying the volume V at pressure P and at temperature T . Therefore,

$$v_{rms}^2 = \frac{3PV}{MN} = \frac{3P}{\rho},$$

where M is the mass of the gas and ρ is its density.



The pressure of the gas is $P = 1.23 \times 10^{-2}$ atm , its density is $\rho = 1.32 \times 10^{-5}$ g cm⁻³. Therefore,

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.23 \times 10^{-2} \times 1.01 \times 10^5}{1.32 \times 10^{-5} \times 10^3}} \text{ m s}^{-1}$$

$$= 5.31 \times 10^2 \text{ m s}^{-1}.$$

One mole of gas has N_A molecules. The total kinetic energy of one mole of gas will therefore be

$$mN_A v_{rms}^2 / 2 = 3kN_A T / 2 = 3RT / 2,$$

and

$$mN_A = \frac{3RT}{v_{rms}^2}.$$

Mass of one mole of the gas will therefore be

$$mN_A = \frac{3RT}{v_{rms}^2} = \frac{3 \times 8.3145 \times (273.16 + 44)}{(5.31 \times 10^2)^2} \text{ kg} = 28.0 \text{ g}.$$

Therefore, the gas is nitrogen (N_2).