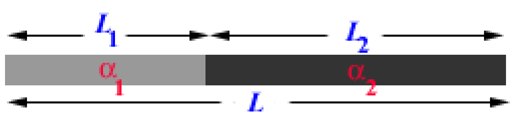
203.

<u>Problem 22.37 (RHK)</u>

A composite bar of length $L = L_1 + L_2$ *is made from a bar of material* 1 *and length* L_1 *attached to a bar of material* 2 *and length* L_2. *(a) We have to show that the effective coefficient of linear expansion* α *for this bar is given by* $\alpha = (\alpha_1 L_1 + \alpha_2 L_2)/L$. *(b) Using steel and brass we have to design such a composite bar whose length is* 52.4 cm *and whose effective coefficient of linear expansion is* $13 \times 10^{-6} / \mathrm{C}^0$.
The coefficients of linear expansion for steel and brass are $\alpha_{steel} = 11 \times 10^{-6} / \mathrm{C}^0$ *and* $\alpha_{brass} = 19 \times 10^{-6} / \mathrm{C}^0$.



<u>Solution:</u>

(a)

When the temperature of the composite material is raised by ΔT C^0, length of the bar of material 1 changes by

$$\Delta L_1 = \alpha_1 L_1 \Delta T,$$

and that of the bar of material 2 changes by

$\Delta L_2 = \alpha_2 L_2 \Delta T$.

The change in length of the composite bar will therefore be

$$\Delta L = \Delta L_1 + \Delta L_2 = \left(\alpha_1 L_1 + \alpha_2 L_2\right)\Delta T = \alpha L \Delta T.$$

Therefore, the effective linear expansion coefficient of the composite bar is

$$\alpha = \left(\alpha_1 L_1 + \alpha_2 L_2\right)/L \; .$$

(b)

Let the bar 1 be steel and its length be L_1 and the bar 2 be brass and its length be L_2. The length of the composite bar L is 52.4 cm. We want the linear coefficient of expansion of the composite bar to be $\alpha = 13\times 10^{-6}/\mathrm{C}^0$. We therefore have

$$13 = \left(11L_1 + 19\left(L - L_1\right)\right)/L.$$

Substituting $L = 52.4$ cm, we find that the length of the steel bar has to be 39.3 cm and that of the brass bar has to be 13.1 cm.