202.

Problem 22.35 (RHK)

At $100\,^0\mathrm{C}$ *a glass flask is filled by* 891 g *of mercury. We have to calculate the mass of mercury that is needed to fill the flask at* $-35\,^0\mathrm{C}$.

The coefficient of linear expansion of glass $\alpha_{glass} = 9.0\times10^{-6}/\mathrm{C}^0$; *and the coefficient of volume expansion of mercury is* $\beta_{mercury} = 1.8\times10^{-4}/\mathrm{C}^0$ *and density of mercury at* $20\,^0\mathrm{C}$,

$\rho_{mercury}\left(20\,^0\mathrm{C}\right) = 13.55\ \mathrm{g/cm^3}$.

Solution:

We will first calculate the density of mercury at $100\,^0\mathrm{C}$. Change in density with temperature is determined by the coefficient of volume expansion.

$$\rho' = \rho + \Delta\rho = \frac{M}{V + \beta V\Delta T} = \rho\left(1+\beta\Delta T\right)^{-1}\ ;\ \ \rho - \beta\rho\Delta T,$$

or

$$\Delta\rho = -\beta\rho\,\Delta T.$$

Therefore,

$$\rho_{mercury}\left(100\,^0\mathrm{C}\right) = \frac{13.55}{1+1.8\times10^{-4}\times80}\ \text{g cm}^{-3}$$
$$= \frac{13.55}{1.0144}\ \text{g cm}^{-3} = 13.357\ \text{g cm}^{-3}.$$

Volume of 891 g of mercury at $100\,^0\mathrm{C}$ will be

$$V_{mercury} = \frac{891}{13.357}\ \text{cm}^3 = 66.703\ \text{cm}^3.$$

Change in the volume of mercury when its temperature changes from $100\,^0\mathrm{C}$ to $-35\,^0\mathrm{C}$ will be

$$\Delta V_{mercury} = -135\times1.8\times10^{-4}\times66.703\ \text{cm}^3 = -1.6208\ \text{cm}^3.$$

We will next calculate the change in the volume of the glass flask when its temperature changes from $100\,^0\mathrm{C}$ to $-35\,^0\mathrm{C}$. It is related to the coefficient of linear expansion α_{glass}.

$$\Delta V_{flask} = 3\times\alpha_{glass}V_{flask}\times\Delta T = -3\times9.0\times10^{-6}\times66.703\times135\ \text{cm}^3$$
$$= -0.243132\ \text{cm}^3.$$

Therefore,

$$V_{flask}\left(-35\,^0\mathrm{C}\right) = \left(66.703-0.243132\right)\ \text{cm}^3 = 66.4598\ \text{cm}^3.$$

The volume of the empty flask at $-35\,^0\mathrm{C}$ will therefore be

$$V_{flask}\left(-35^{0}\text{C}\right)-V_{mercury}\left(-35^{0}\text{C}\right)=\left(66.4598-65.0822\right)\ \text{cm}^{3}$$
$$=1.3776\ \text{cm}^{3}.$$

We will next calculate the density of mercury at -35^{0}C.

It will be

$$\rho_{mercury}\left(-35^{0}\text{C}\right)=\frac{\rho_{mercury}\left(20^{0}\text{C}\right)}{1-1.8\times10^{-4}\times55}=\frac{13.55}{0.9901}\ \text{g cm}^{-3}$$
$$=13.685\ \text{g cm}^{-3}.$$

Additional amount of mercury needed to fill the flask at -35^{0}C will be $1.3776\times13.685\ \text{g}=18.85$ g.

Therefore, the total amount of mercury in the flask at -35^{0}C is

$\left(891+18.85\right)\ \text{g}=909.8$ g.