

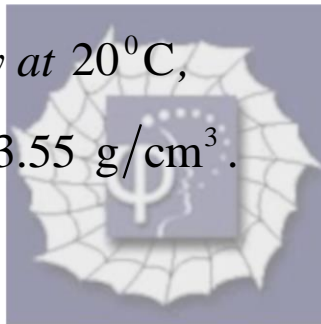
202.

Problem 22.35 (RHK)

At 100⁰C a glass flask is filled by 891 g of mercury. We have to calculate the mass of mercury that is needed to fill the flask at -35⁰C.

The coefficient of linear expansion of glass $\alpha_{\text{glass}} = 9.0 \times 10^{-6} / \text{C}^0$; and the coefficient of volume expansion of mercury is $\beta_{\text{mercury}} = 1.8 \times 10^{-4} / \text{C}^0$ and density of mercury at 20⁰C,

$$\rho_{\text{mercury}}(20^0\text{C}) = 13.55 \text{ g/cm}^3.$$



Solution:

We will first calculate the density of mercury at 100⁰C.

Change in density with temperature is determined by the coefficient of volume expansion.

$$\rho' = \rho + \Delta\rho = \frac{M}{V + \beta V \Delta T} = \rho(1 + \beta \Delta T)^{-1}; \quad \rho - \beta \rho \Delta T,$$

or

$$\Delta\rho = -\beta\rho \Delta T.$$

Therefore,

$$\begin{aligned}\rho_{\text{mercury}}(100^{\circ}\text{C}) &= \frac{13.55}{1 + 1.8 \times 10^{-4} \times 80} \text{ g cm}^{-3} \\ &= \frac{13.55}{1.0144} \text{ g cm}^{-3} = 13.357 \text{ g cm}^{-3}.\end{aligned}$$

Volume of 891 g of mercury at 100°C will be

$$V_{\text{mercury}} = \frac{891}{13.357} \text{ cm}^3 = 66.703 \text{ cm}^3.$$

Change in the volume of mercury when its temperature changes from 100°C to -35°C will be

$$\Delta V_{\text{mercury}} = -135 \times 1.8 \times 10^{-4} \times 66.703 \text{ cm}^3 = -1.6208 \text{ cm}^3.$$

We will next calculate the change in the volume of the glass flask when its temperature changes from 100°C to -35°C . It is related to the coefficient of linear expansion

α_{glass} .

$$\begin{aligned}\Delta V_{\text{flask}} &= 3 \times \alpha_{\text{glass}} V_{\text{flask}} \times \Delta T = -3 \times 9.0 \times 10^{-6} \times 66.703 \times 135 \text{ cm}^3 \\ &= -0.243132 \text{ cm}^3.\end{aligned}$$

Therefore,

$$V_{\text{flask}}(-35^{\circ}\text{C}) = (66.703 - 0.243132) \text{ cm}^3 = 66.4598 \text{ cm}^3.$$

The volume of the empty flask at -35°C will therefore be

$$V_{flask}(-35^{\circ}\text{C}) - V_{mercury}(-35^{\circ}\text{C}) = (66.4598 - 65.0822) \text{ cm}^3 \\ = 1.3776 \text{ cm}^3.$$

We will next calculate the density of mercury at -35°C .

It will be

$$\rho_{mercury}(-35^{\circ}\text{C}) = \frac{\rho_{mercury}(20^{\circ}\text{C})}{1 - 1.8 \times 10^{-4} \times 55} = \frac{13.55}{0.9901} \text{ g cm}^{-3} \\ = 13.685 \text{ g cm}^{-3}.$$

Additional amount of mercury needed to fill the flask at -35°C will be $1.3776 \times 13.685 \text{ g} = 18.85 \text{ g}$.

Therefore, the total amount of mercury in the flask at -35°C is

$$(891 + 18.85) \text{ g} = 909.8 \text{ g}.$$