201.

Problem 22.41 (RHK)

A pendulum clock with a pendulum made of brass is designed to keep accurate time at 20^0C. We have to estimate the error in seconds per hour when the clock operates at 0^0C.

$\alpha_{brass} = 19 \times 10^{-6} / \text{C}^0$.

Solution:

We will first find the length of the pendulum for it to have a period of 1 s when it operates at 20^0C. The period of the pendulum is related to its length, *l*, and the acceleration due to gravity, *g*, as

$$T = 2\pi\sqrt{\frac{l}{g}} \ ,$$

or

$$l = \frac{T^2 g}{4\pi^2} \ .$$

For $T = 1$ s,

$$l = \frac{9.8}{4\pi^2} \text{ m} = 0.248237 \text{ m} \ .$$

Change in length of the brass second-pendulum when it operates at 0^0C, will be

$$\Delta l = -0.248237 \times 19 \times 10^{-6} \times 20 \text{ m}$$
$$= -0.000094 \text{ m}.$$

The length of the pendulum of the clock at 0^0C will therefore be

$$l' = l - \Delta l = (0.248237 - 0.000094) \text{ m}$$
$$= 0.2481429 \text{ m}.$$

This clock will show 1-second lapse after each period of the pendulum. The period of oscillation of the pendulum at 0^0C will be

$$T' = 2\pi\sqrt{\frac{0.2481429}{9.8}} \text{ s} = 0.9998106 \text{ s}.$$

That is when time lapse is 0.9998106 s the clock shows that 1 s has lapsed.

Therefore, error in 1 s is

$$= (1.0 - 0.9998106) \text{ s} = 1.894 \times 10^{-4} \text{ s}.$$

Error in 1 hour will be $1.894 \times 10^{-4} \times 3600 \text{ s} = 0.68 \text{ s}$.