

201.

Problem 22.41 (RHK)

A pendulum clock with a pendulum made of brass is designed to keep accurate time at 20°C . We have to estimate the error in seconds per hour when the clock operates at 0°C .

$$\alpha_{\text{brass}} = 19 \times 10^{-6} / \text{C}^{\circ}.$$

Solution:

We will first find the length of the pendulum for it to have a period of 1 s when it operates at 20°C . The period of the pendulum is related to its length, l , and the acceleration due to gravity, g , as

$$T = 2\pi \sqrt{\frac{l}{g}},$$

or

$$l = \frac{T^2 g}{4\pi^2}.$$

For $T = 1$ s,

$$l = \frac{9.8}{4\pi^2} \text{ m} = 0.248237 \text{ m}.$$

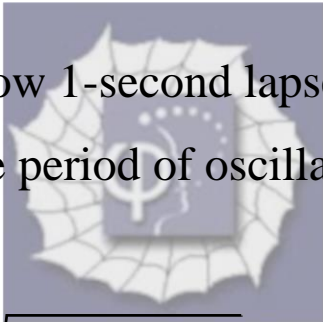
Change in length of the brass second-pendulum when it operates at 0°C , will be

$$\begin{aligned}\Delta l &= -0.248237 \times 19 \times 10^{-6} \times 20 \text{ m} \\ &= -0.000094 \text{ m} .\end{aligned}$$

The length of the pendulum of the clock at 0°C will therefore be

$$\begin{aligned}l' &= l - \Delta l = (0.248237 - 0.000094) \text{ m} \\ &= 0.2481429 \text{ m} .\end{aligned}$$

This clock will show 1-second lapse after each period of the pendulum. The period of oscillation of the pendulum at 0°C will be


$$T' = 2\pi \sqrt{\frac{0.2481429}{9.8}} \text{ s} = 0.9998106 \text{ s} .$$

That is when time lapse is 0.9998106 s the clock shows that 1 s has lapsed.

Therefore, error in 1 s is

$$= (1.0 - 0.9998106) \text{ s} = 1.894 \times 10^{-4} \text{ s} .$$

Error in 1 hour will be $1.894 \times 10^{-4} \times 3600 \text{ s} = 0.68 \text{ s} .$