200.

<u>Problem 22.45 (RHK)</u>

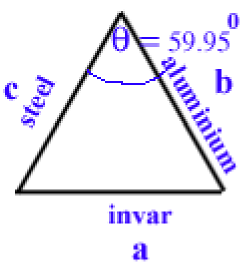
Three equal-length straight rods, of aluminium, invar, and steel, all at 20^0C, *form an equilateral triangle with hinge pins at the vertices. We have to find the temperature at which the angle opposite the invar rod will become* 59.95^0.

The linear thermal coefficients of these materials are

$\alpha_{invar} = 0.7\times10^{-6}/\text{C}^0$, $\alpha_{steel} = 11\times10^{-6}/\text{C}^0$ *and*

$\alpha_{Al} = 23\times10^{-6}/\text{C}^0$.

<u>Solution:</u>



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Let l be the length of the rods at 20^0C. When the temperature of the rods becomes $20^0\text{C} + \Delta T\ \text{C}^0$ the changes in the lengths of the rods will be determined by the coefficients of their linear expansion. We have

$\Delta l_{invar} = l\alpha_{invar}\Delta T$,

$\Delta l_{steel} = l\alpha_{steel}\Delta T$,

and

$\Delta_{Al} = l\alpha_{Al}\Delta T$.

Using the trigonometric property of a triangle that

$a^2 = b^2 + c^2 - 2bc\cos\theta$,

we have the relation,

$$\left(1+\alpha_{invar}\Delta T\right)^2 = \left(1+\alpha_{steel}\Delta T\right)^2 + \left(1+\alpha_{Al}\Delta T\right)^2 - 2\left(1+\alpha_{steel}\Delta T\right)\left(1+\alpha_{Al}\Delta T\right)\cos 59.95^0 \;.$$

Neglecting terms of order $\alpha^2\left(\Delta T\right)^2$, we get

$$1+2\alpha_{invar}\Delta T = 1+2\alpha_{steel}\Delta T + 1+2\alpha_{Al}\Delta T - 2\left(1+\alpha_{steel}\Delta T + \alpha_{Al}\Delta T\right)\cos 59.95^0.$$

Substituting the values of α_{invar}, α_{Al}, and α_{steel}, we solve the above equation for ΔT. We find

$\Delta T = 46.4\ \mathrm{C}^0$.

Therefore, the temperature at which the angle opposite the invar bar will become 59.95^0 will be $66.4^0\mathrm{C}$.