199.

Problem 22.43 (RHK)

A 1.28-m-long vertical glass tube is half-filled with a liquid at 20.0^0C. *We have to find the change in height of the liquid column when the tube is heated to* 33.0^0C. *We may assume that* $\alpha_{glass} = 1.1\times10^{-5}/\text{C}^0$ *and* $\beta_{liquid} = 4.2\times10^{-5}/\text{C}^0$.

Solution:

Let A be the cross-sectional area of the tube. The volume of the liquid, V_{liquid}, at 20.0^0C will be

$$V_{liquid} = A\times 0.64 \text{ m}^3.$$

The change in the volume of the liquid when it is heated to 33.0^0C will be

$$\begin{aligned}\Delta V_{liquid} &= \beta V_{liquid}\Delta T \\ &= 4.2\times10^{-5}\times 0.64A\times 13 \text{ m}^3 \\ &= 34.94\times10^{-5} A \text{ m}^3.\end{aligned}$$

Therefore, the volume of the liquid at 33.0^0C will be

$$\begin{aligned}V_{liquid}\left(33.0^0\text{C}\right) &= A\left(0.64 + 34.94\times10^{-5}\right) \text{ m}^3 \\ &= 0.64035A \text{ m}^3.\end{aligned}$$

The cross-sectional area of the glass tube would also increase due to thermal expansion when the tube is heated to 33.0^0C from 20.0^0C. The change in the coss-sectional area will be

$$\Delta A = 2\alpha_{glass}\Delta T$$

$$=2\times 1.1\times 10^{-5}\times 13\times A \text{ m}^2$$

$$= 0.000286A \text{ m}^2.$$

And

$$A + \Delta A = 1.0000286A \text{ m}^2.$$

Let the height of the liquid column in the glass tube at 33.0^0C be h m.

Then

$$(1.000286)Ah = 0.64305A$$

or

$$h = \frac{0.64035}{1.000286} \text{ m} = 0.64017 \text{ m}.$$

Therefore, the change in height of the liquid in the glass tube will be

$$h(33^0\text{C}) - h(20^0\text{C}) = (0.64017 - 0.64) \text{ m}$$

$$= 0.17 \text{ mm}.$$