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Problem 22.47 (RHK)

An aluminium cube 20 cm *on edge floats on mercury. We have to find how much farther will the block sink when the temperature rises from* 270 to 320 K. (*The coefficient of volume expansion of mercury is* $1.8\times10^{-4}/\mathrm{C}^0$.)

Solution:

In answering this problem we will use the property of volume expansion of mercury and determine its density at 270 K and at 320 K from its density at 293 K, $\rho_{Hg} = 2.699\ \mathrm{g/cm^3}$. We expect that the aluminium block will sink further in mercury as the temperature of mercury changes from 270 K to 320 K because of the change in density.

The coefficient of volume expansion is defined as

$$\beta = \frac{\Delta V/V}{\Delta T}.$$

Therefore, change in density with change ΔT in temp will be

$\Delta\rho = -\beta\rho\,\Delta T$.

We thus find

$$\rho_{Hg}\left(270K\right) = \left(1+\beta\times 23\right)\times\rho_{Hg}\left(293K\right)$$

$$= \left(1+1.8\times 10^{-4}\times 23\right)\times 13.55 \text{ g cm}^{-3}$$

$$= 13.606 \text{ g cm}^{-3}.$$

And

$$\rho_{Hg}\left(320\text{K}\right) = \left(1-1.8\times 10^{-4}\times 50\right)\times\rho_{Hg}\left(270\text{ K}\right)$$

$$= 13.484 \text{ g cm}^{-3}.$$

We use for the density of aluminium, $\rho_{Al} = 2.699 \text{ g cm}^{-3}$.

Therefore, the mass of aluminium cube of edge 20 cm is

$$M = 20^3 \times 2.699 \text{ g} = 21{,}592 \text{ g}.$$

The height, *h*, of the aluminium block inside the mercury at 270 K can be determined from the buoyancy principle. It will be

$$20^2\left(\text{cm}^2\right)\times h\times 13.606\left(\text{g cm}^{-3}\right) = 21592\left(\text{g}\right),$$

or

$$h = 3.967 \text{ cm}.$$

When the aluminium block is placed in mercury at 320 K the area of its base would have expanded because of thermal expansion. The coefficient of thermal expansion of aluminium is $23\times10^{-6}/\mathrm{C}^0$. Therefore, the change in area of the base of the aluminium cube will be

$$\Delta A = 2\alpha A\Delta T = 2\times 23\times10^{-6}\times 400\times 50\ \mathrm{cm}^2$$
$$= 0.920\ \mathrm{cm}^2\ .$$

The area of the base of the aluminium block will be

$$A + \Delta A = 400.92\ \mathrm{cm}^2.$$

Therefore, the height, h', of the aluminium cube inside the mercury at 320 K can be determined by using the density of mercury at 320 K. It will be

$$h'\times 400.92\left(\mathrm{cm}^2\right)\times 13.484\left(\mathrm{g\ cm}^{-3}\right) = 21{,}592\ \mathrm{g}$$

or

$$h' = 3.9940\ \mathrm{cm}.$$

Therefore, the additional height by which the aluminium block will sink inside mercury with the change in temperature from 270 K to 320 K will be

$$\Delta h = h' - h = \left(3.9940 - 3.967\right)\ \mathrm{cm} = 0.27\ \mathrm{mm}.$$