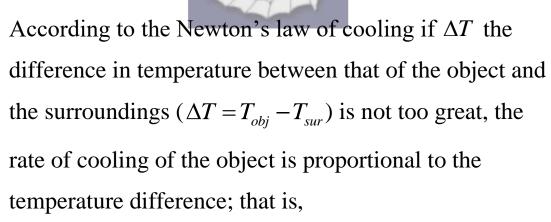
196.

Problem 22.10 (RHK)

Early in the morning the heater of a house breaks down. The outside temperature is -7.0° C. As a result, the inside temperature drops from 22 to 18° C in 45 min. We have to find how much longer will it take for the inside temperature to fall by another 4.0° C. We can assume that the outside temperature does not change and that the Newton's law of cooling applies.

Solution:



$$\frac{d\Delta T}{dt} = -A(\Delta T),$$

where *A* is a constant. On integrating the above differential equation, we express its solution as

$$\Delta T = \Delta T_0 e^{-A}$$

at a time t later and ΔT_0 is the temperature difference between the object and the surroundings at t = 0. In our problem the temperature of the surroundings, the outside temperature, $T_{sur} = -7.0^{\circ}$ C.

The temperature of the object, that is that of inside of the room, at t = 0 is $T_{obj}(t = 0) = 22^{\circ}$ C. Therefore,

$$\Delta T_0 = (22 - (-7.0))^0 C = 29^0 C.$$

It is given that the inside temperature drops to 18° C in 45 min. That is after 45 min $\Delta T = (18 - (-7))^{\circ}$ C=25.0°C.

We use this data for calculating the constant A.

$$25 = 29e^{-45A}$$
,
or
 $\ln\left(\frac{25}{29}\right) = -45A$,
or
 $A = 3.298 \times 10^{-3}$.

We will next find the additional time t for the inside temperature of the room 18 to 14° C. From the Newton's law of cooling, we have

21 = 25
$$e^{-At}$$
,
or
 $t = -\ln\left(\frac{21}{25}\right) / A = \frac{0.174}{3.298 \times 10^{-3}} \text{min} = 52.8 \text{ min.}$

