196.

Problem 22.10 (RHK)

Early in the morning the heater of a house breaks down. The outside temperature is $-7.0^{0}\mathrm{C}$. *As a result, the inside temperature drops from* 22 *to* $18^{0}\mathrm{C}$ *in* 45 min. *We have to find how much longer will it take for the inside temperature to fall by another* $4.0^{0}\mathrm{C}$. *We can assume that the outside temperature does not change and that the Newton's law of cooling applies.*

Solution:

According to the Newton's law of cooling if ΔT the difference in temperature between that of the object and the surroundings ($\Delta T = T_{obj} - T_{sur}$) is not too great, the rate of cooling of the object is proportional to the temperature difference; that is,

$$\frac{d\Delta T}{dt} = -A\left(\Delta T\right),$$

where A is a constant. On integrating the above differential equation, we express its solution as

$$\Delta T = \Delta T_0 e^{-At}$$

at a time *t* later and ΔT_0 is the temperature difference between the object and the surroundings at $t=0$.

In our problem the temperature of the surroundings, the outside temperature, $T_{sur} = -7.0\,^0\mathrm{C}$.

The temperature of the object, that is that of inside of the room, at $t=0$ is $T_{obj}(t=0)=22\,^0\mathrm{C}$. Therefore,

$$\Delta T_0 = \left(22-(-7.0)\right)^0\mathrm{C} = 29\,^0\mathrm{C}.$$

It is given that the inside temperature drops to $18\,^0\mathrm{C}$ in 45 min. That is after 45 min

$\Delta T = \left(18-(-7)\right)^0\mathrm{C} = 25.0\,^0\mathrm{C}.$

We use this data for calculating the constant *A*.

$25 = 29e^{-45A}$,

or

$\ln\left(\frac{25}{29}\right) = -45A,$

or

$A = 3.298\times10^{-3}.$

We will next find the additional time *t* for the inside temperature of the room 18 to $14\,^0\mathrm{C}$. From the Newton's law of cooling, we have

$$21 = 25e^{-At},$$

or

$$t = -\ln\left(\frac{21}{25}\right)\Big/A = \frac{0.174}{3.298\times10^{-3}}\ \text{min} = 52.8\ \text{min}.$$