922.

<u>Problem 56.29 (RHK)</u>

Due to the presence everywhere of the microwave radiation background, the minimum temperature possible of a gas in interstellar or intergalactic space is not 0 K *but* 2.7 K. *This implies that a significant fraction of the molecules in space that possess excited states of low excitation energy may, in fact, be in those excited states. Subsequent de-excitation leads to the emission of radiation that could be detected. We consider a* (*hypothetical*) *molecule with just one excited state.* (a) *We have to find the energy of the excited state so that* 23% *of the molecules are in the excited state.* (b) *We have to find the wavelength of the photon emitted in a transition to the ground state.*

<u>Solution:</u>

(a)

We consider an ensemble of molecules of a given type with low excitation energy at temperature $T = 2.7$ K. We have to find the energy of the excited state so that 23% of the molecules are in the excited state. Let N_0 be the number of molecules in the ground state of the molecule and let N_1 be the number of molecules in the first excited state of the molecule. Using the Boltzmann law we have

$$\frac{N_1}{N_0} = \frac{23}{77} = \exp\left(-\left(E_1 - E_0\right)/kT\right),\ T = 2.7\ \text{K},$$

or

$$\left(E_1 - E_0\right)/2.7\,k = \ln\left(77/23\right),$$

or

$$\left(E_1 - E_0\right) = 1.208 \times 1.380 \times 10^{-23} \times 2.7\ \text{J}$$

$$= \frac{1.208 \times 1.380 \times 10^{-23} \times 2.7}{1.6 \times 10^{-19}}\ \text{eV} = 2.81 \times 10^{-4}\ \text{eV}$$

$$= 281\ \mu\text{eV}.$$

(b)

The wavelength of the photon emitted in transition from the first excited state to the ground state of the molecule will be

$$\lambda = \frac{hc}{\left(E_1 - E_0\right)} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}\ \text{J m}}{2.81 \times 10^{-4}\ \text{eV}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2.81 \times 10^{-4} \times 1.6 \times 10^{-19}}\ \text{m}$$

$$= 4.42 \times 10^{-3}\ \text{m} = 4.42\ \text{mm}.$$