921.

<u>Problem 56.28 (RHK)</u>

The recessional speeds of galaxies and quarks at great distances are close to the speed of light, so that relativistic Doppler shift formula must be used. The red shift is reported as z, $z = \Delta\lambda/\lambda_0$ *is the* (*fractional*) *red shift*. (a) *We have to show that, in terms of z, the recessional speed parameter* $\beta = v/c$ *is given by*

$$\beta = \frac{z^2 + 2z}{z^2 + 2z + 2}.$$

(b) *The most distant quasar detected* (*as of* 1990) *has* $z = 4.43$. *We have to calculate its speed parameter*. (c) *We have to find the distance to the quasar, assuming that the Hubble's law is valid to these distances*.

<u>Solution:</u>

For relativistic speeds the Doppler shift is given by the formula

$$\lambda = \lambda_0\sqrt{\frac{(1+v/c)}{(1-v/c)}} = \lambda_0\sqrt{\frac{(1+\beta)}{(1-\beta)}},.$$

where $\beta = v/c$.

Therefore, the fractional red shift is related to β by the equation

$$z=\frac{\lambda-\lambda_0}{\lambda_0}=\sqrt{\frac{(1+\beta)}{(1-\beta)}}-1,$$

or

$$(1+z)^2=\frac{(1+\beta)}{(1-\beta)},$$

or

$$\left(1+z^2+2z\right)\times(1-\beta)=(1+\beta),$$

or

$$\left(2+z^2+2z\right)\beta=z^2+2z,$$

or

$$\beta=\frac{z^2+2z}{\left(2+z^2+2z\right)}.$$

(b)

The most distant quasar detected (as of 1990) has $z=4.43$. Its recessional speed parameter β will therefore be

$$\beta=\frac{z^2+2z}{z^2+2z+2}=\frac{4.43^2+2\times4.43}{4.43^2+2\times4.43+2}=\frac{28.48}{30.48}$$
$$=0.9345.$$

(c)

The Hubble's law is

$v = Hd,$

$$H = 67\,\frac{\text{km/s}}{\text{Mpc}},$$

$$\begin{aligned} 1\ \text{Mpc} &= 3.26 \times 10^{6}\ \text{light-years} \\ &= 3.084 \times 10^{19}\ \text{km.} \end{aligned}$$

Therefore, the distance to the quasar for which the fractional red shift is 4.43, or the equivalent recessional speed is 0.9345 *c*, will be

$$d = \frac{v}{H} = \frac{0.9345 \times 3 \times 10^{5}\ \text{km s}^{-1}}{67\ \text{km s}^{-1}}\ \text{Mpc}$$

$$= 4.18 \times 10^{3}\ \text{Mpc} = 4.18 \times 10^{3} \times 3.26 \times 10^{6}\ \text{light-years}$$

$$= 13.6 \times 10^{9}\ \text{light-years.}$$