919.

Problem 56.26 (RHK)

In the laboratory, one of the lines of sodium is emitted at a wavelength of 590.0 nm. *When observing this light from a particular galaxy, however, this line is seen at a wavelength of* 602.0 nm. *We have to calculate the distance to the galaxy, assuming that Hubble's law holds.*

Solution:

The relativistic Doppler shift formula is

$$\lambda = \lambda_0 \sqrt{\frac{(1+v/c)}{(1-v/c)}}\,.$$

Light of wavelength 590.0 nm is observed to be 602.0 nm. Therefore, the speed of the source can be obtained from the equation

$$\sqrt{\frac{(1+v/c)}{(1-v/c)}} = \frac{\lambda}{\lambda_0} = \frac{602.0\ \text{nm}}{590.0\ \text{nm}} = 1.0203,$$

or

$$\frac{(1+v/c)}{(1-v/c)} = 1.04109,$$

or

$$1 + v/c = 1.04109 - 1.04109\, v/c,$$

or

$$v/c = \frac{0.04109}{2.04109} = 2.013 \times 10^{-2}.$$

The Hubble's law is

$$v = Hd,$$

$$H = 67 \frac{\text{km/s}}{\text{Mpc}},$$

$$1\ \text{Mpc} = 3.26 \times 10^{6}\ \text{light-years}$$

$$= 3.084 \times 10^{19}\ \text{km}.$$

Therefore, the distance of the galaxy as calculated using the Hubble's law will be

$$d = \frac{v}{H} = \frac{2.013 \times 10^{-2} \times 3 \times 10^{5}\ \text{km s}^{-1}}{67\ \text{km s}^{-1}}\ \text{Mpc}$$

$$= 90.1\ \text{Mpc}.$$