

**918.**

**Problem 56.25 (RHK)**

*We have to find the observed wavelength of the 656.3-nm  $H_\alpha$  line of hydrogen emitted by a galaxy at a distance of  $2.4 \times 10^8$  pc.*

**Solution:**

We assume that the Hubble's law is applicable in calculating the receding speed of the galaxy, which is at a distance of  $2.4 \times 10^8$  pc.

Hubble's law is

$$v = Hd,$$

$$H = 67 \frac{\text{km/s}}{\text{Mpc}},$$

$$\begin{aligned} 1 \text{ Mpc} &= 3.26 \times 10^6 \text{ light-years} \\ &= 3.084 \times 10^{19} \text{ km.} \end{aligned}$$

Therefore, the recessional speed of the galaxy at a distance of  $2.4 \times 10^8$  pc will be

$$v = 67 \times 2.4 \times 10^2 \text{ km s}^{-1} = 160.8 \times 10^5 \text{ m s}^{-1}.$$

For relativistic speeds the Doppler shift can be obtained from the formula

$$\lambda = \lambda_0 \sqrt{\frac{(1+v/c)}{(1-v/c)}}.$$

The wavelength of the  $H_\alpha$  line of hydrogen,

$\lambda_0 = 656.3$  nm, emitted by a galaxy at a distance of  $2.4 \times 10^8$  pc when observed from the Earth will be

$$\begin{aligned} \lambda &= 656.3 \sqrt{\frac{1+160.8 \times 10^5 / 3 \times 10^8}{1-160.8 \times 10^5 / 3 \times 10^8}} \text{ nm} \\ &= 656.3 \sqrt{\frac{1+0.0536}{1-0.0536}} \text{ nm} = 692.5 \text{ nm}. \end{aligned}$$

