910.

Problem 45.11P (HRW)

A stationary particle m_0 *decays into two particles* m_1 *and* m_2, *which move off with equal but oppositely directed momenta. We have to show that the kinetic energy* K_1 *of* m_1 *is given by*

$$K_1 = \frac{1}{2E_0}\left[\left(E_0 - E_1\right)^2 - E_2^2\right],$$

where m_0, m_1 *and* m_2 *are masses and* E_0, E_1 and E_2 *are the corresponding rest energies.*

Solution:

Let the magnitude of momentum of the final particles be p. From conservation of energy, we note that the sum of relativistic energies of the final particles m_1 and m_2 has to be equal to the rest mass energy of the initial particle m_0.

We thus have the equation

$$E_0 = \left(p^2c^2 + E_1^2\right)^{1/2} + \left(p^2c^2 + E_2^2\right)^{1/2}.$$

Or,

$$E_0 - \left(p^2c^2 + E_1^2\right)^{1/2} = +\left(p^2c^2 + E_2^2\right)^{1/2}.$$

We square the above equation and carry out algebraic simplifications. We get

$$\left(p^2c^2 + E_1^2\right)^{1/2} = \frac{E_0^2 + E_1^2 - E_2^2}{2E_0}.$$

The kinetic energy of the particle m_1 will therefore be given by

$$K_1 = \left(p^2c^2 + E_1^2\right)^{1/2} - E_1 = \frac{E_0^2 + E_1^2 - E_2^2}{2E_0} - E_1$$

$$= \frac{\left(E_0 - E_1\right)^2 - E_2^2}{2E_0}.$$