905.

Problem 56.7 (RHK)

A neutral pion has a rest energy of 135 MeV *and a mean life of* 8.4×10^{-17} s. *It is produced with an initial kinetic energy of* 80 MeV *and it decays after one mean lifetime. We have to find the longest possible track that this particle could leave in a bubble chamber.*

Solution:

The kinetic energy K, the rest mass energy mc^2 and the Lorentz factor $\gamma = 1/\sqrt{1-(v/c)^2}$ are related as

$$K + mc^2 = \frac{mc^2}{\sqrt{1-(v/c)^2}} = mc^2\gamma,$$

$$\gamma = \left(1 + \frac{K}{mc^2}\right).$$

Therefore, the Lorentz factor γ of the pion of rest mass energy 135 MeV and kinetic energy of 80 MeV will be

$$\gamma = 1 + \frac{80}{135} = 1.5926.$$

The speed of the pion will therefore be

$$\frac{v}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = \sqrt{\frac{1.5926^2 - 1}{1.5926^2}} = 0.778,$$

or

$$v = 0.778 \times 3 \times 10^8 \text{ m s}^{-1} = 2.335 \times 10^8 \text{ m s}^{-1}.$$

The mean lifetime of neutral pion measured in its rest frame is 8.4×10^{-17} s. Therefore, the lifetime measured in the laboratory frame with speed v will be given by the relativistic formula

$$t = t_0 \times \gamma = 8.4 \times 10^{-17} \times 1.5926 \text{ s}$$
$$= 1.337 \times 10^{-16} \text{ s}.$$

Therefore, the longest possible track that this particle can leave in a bubble chamber will be

$$l = vt = 2.335 \times 10^8 \times 1.337 \times 10^{-16} \text{ m}$$
$$= 3.12 \times 10^{-8} \text{ m} = 31 \text{ nm}.$$