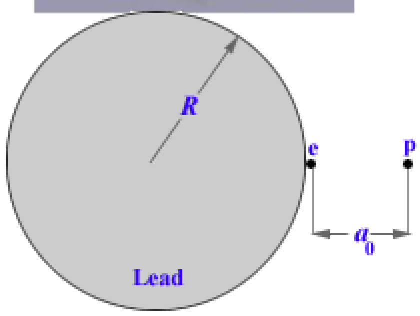
903.

Problem 56.3 (RHK)

An electron and a proton are placed a distance apart equal to one Bohr radius a_0*. We have to find the radius R of a lead sphere that must be placed directly behind the electron so that the gravitational force on the electron just overcomes the electrostatic attraction between the proton and the electron; see the figure. We may assume that the Newton's law of gravitation holds, and that the density of the sphere equals the density of the lead on Earth.*



Solution:

The magnitude of Coulomb force of attraction between the electron and the proton separated by one Bohr radius will be given by

$$F_C = \frac{e^2}{4\pi\varepsilon_0 a_0{}^2},$$

where $a_0 = 5.29\times10^{-11}$ m. We have

$$F_C = \frac{e^2}{4\pi\varepsilon_0 a_0{}^2} = \frac{8.99\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(5.29\times10^{-11}\right)^2}\ \text{N}$$

$$= 8.22\times10^{-8}\ \text{N}.$$

Let the radius of the lead sphere be R. The density of lead on Earth is $\rho_{\text{lead}} = 11.36\times10^3$ kg m^{-3}. The magnitude of the force of gravitational attraction on the electron due to the mass of the spherical lead ball of radius R will be

$$F_g = \frac{G\times\left(4\pi/3\right)R^3\times\rho_{\text{lead}}\times m_{\text{e}}}{R^2}$$

$$= \frac{4\pi G\rho_{\text{lead}}m_{\text{e}}}{3}R = \frac{4\pi}{3}\times6.67\times10^{-11}\times11.36\times10^3\times9.11\times10^{-31}\times R\ \text{N}$$

$$= 2.89\times10^{-36}R\ \text{N m}^{-1}.$$

Therefore, the radius R of a lead sphere that must be placed directly behind the electron so that the gravitational force on the electron just overcomes the electrostatic attraction between the proton and the electron will be given by requiring

$F_C = F_g$,

or

$2.89 \times 10^{-36} R$ N m^{-1}=8.22×10^{-8} N,

or

$$R = \frac{8.22 \times 10^{-8}}{2.89 \times 10^{-36}} \text{ m} = 2.84 \times 10^{28} \text{ m}.$$