901.

Problem 55.58 (RHK)

Suppose that a fuel pellet in a laser fusion device is made of a liquid deuterium-tritium mixture containing equal numbers of deuterium and tritium atoms. The density of the uncompressed pellet is $200\ \mathrm{kg\ m^{-3}}$ *and the uncompressed radius of the fuel pellet is* $200\ \mu\mathrm{m}$. *Suppose that the compressed fuel pellet "burns" with an efficiency of* 10%. *That is, only* 10% of *the deuterons and* 10% *of the tritons participate in the fusion reaction*

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \mathrm{n} \quad \left(Q = 17.59\ \mathrm{MeV}\right).$$

We have to calculate (a) *the amount of energy is that would be released when the pellet is compressed with lasers so that fusion takes place*. (b) *We have to express this energy in equivalent of* TNT. *The heat of combustion of* TNT *is* $4.6\ \mathrm{MJ\ kg^{-1}}$. (c) *If a fusion reactor is constructed on the basis of* 100 *micro-explosions per second of pellets, we have to calculate the power of the reactor*. (*Note that part of this power must be used to operate the lasers*.)

Solution:

(a)

Let the number of deuterium or the equal number of tritium atoms per 1 m^3 be *n*. The density of the uncompressed deuterium-tritium mixture containing equal numbers of deuterium and tritium atoms is $200\ \mathrm{kg\ m^{-3}}$. Let mass of a deuterium atom be $m_{^2\mathrm{H}}$ and mass of a tritium atom be $m_{^3\mathrm{H}}$. We thus have

$$n\left(m_{^2\mathrm{H}} + m_{^3\mathrm{H}}\right) = 200\ \mathrm{kg\ m^{-3}},$$

or

$$n\left(\frac{(2+3)\times10^{-3}\ \mathrm{kg}}{6.02\times10^{23}}\right) = 200\ \mathrm{kg\ m^{-3}},$$

or

$$n = \frac{200\times6.02\times10^{23}}{5\times10^{-3}}\ \mathrm{m^{-3}} = 2.41\times10^{28}\ \mathrm{m^{-3}}.$$

The radius of a fuel pellet is $200\ \mu\mathrm{m}$. Therefore, the total number of $^2\mathrm{H}$ atoms (or equal number of $^3\mathrm{H}$ atoms) contained in the pellet will be

$$N_{^2\mathrm{H}} = \frac{4\pi\left(200\times10^{-6}\ \mathrm{m}\right)^3\times2.41\times10^{28}\ \mathrm{m^{-3}}}{3}$$

$$= 8.08\times10^{17}.$$

We assume that deuterons and tritons fuse with each other as per the process

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \mathrm{n}.$$

The Q value for this process is 17.59 MeV. Assuming that the compressed fuel pellet "burns" with an efficiency of 10%, the energy released due to fusion in each pellet will therefore be

$$E = 8.08\times10^{17}\times0.10\times17.59\ \mathrm{MeV}$$
$$= 1.42\times10^{18}\ \mathrm{MeV} = 1.42\times10^{18}\times1.6\times10^{-13}\ \mathrm{J}$$
$$= 227\ \mathrm{kJ}.$$

(b)

The amount of fusion energy per pellet in unit of TNT will be

$$E = \frac{1.0\ \text{kg of TNT}\times227\times10^{3}\ \mathrm{J}}{4.6\times10^{6}\ \mathrm{J}} = 49.3\times10^{-3}\ \text{kg of TNT}$$
$$= 49.3\ \text{g of TNT}.$$

(c)

If the fusion reactor is constructed on the basis of 100 micro-explosions per second of pellets, the power generated in the reactor will be

$$P = 227\times100\ \mathrm{kJ\ s^{-1}} = 22.7\ \mathrm{MW}.$$