900.

Problem 55.57 (RHK)

Assume that a plasma temperature of 1.3×10^{8} K *is reached in a laser-fusion device*. (a) *We have to find the most probable speed of a deuteron at this temperature*; (b) *and calculate the distance such a deuteron would move in the confinement time of* 2×10^{-12} s.

Solution:

(a)

The Maxwellian speed distribution, for a classical gas at temperature T and containing N molecules, is

$$n(v)=4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^{2}e^{-mv^{2}/2kT}.$$

The most probable speed is the speed at which $n(v)$ has its maximum value. We find it by requiring that

$dn(v)/dv=0$.

That is

$$\left(2v - v^2 \times \frac{mv}{kT}\right) e^{-mv^2/2kT} = 0,$$

or

$$v_p = \sqrt{\frac{2kT}{m}}.$$

The mass of a deuteron atom will be

$$m_{^2\mathrm{H}} = \frac{2.0\times10^{-3}\ \mathrm{kg}}{6.02\times10^{23}} = 3.32\times10^{-27}\ \mathrm{kg}.$$

Therefore, the most probable speed of a deuteron at

$1.3\times10^{8}\ \mathrm{K}$ will be

$$\left(v_{^2\mathrm{H}}\right)_p = \sqrt{\frac{2kT}{m_{^2\mathrm{H}}}} = \sqrt{\frac{2\times1.38\times10^{-23}\times1.3\times10^{8}}{3.32\times10^{-27}}}\ \mathrm{m\ s^{-1}}$$

$$= 1.04\times10^{6}\ \mathrm{m\ s^{-1}}.$$

(b)

The distance a deuteron having a speed of

$1.04\times10^{6}\ \mathrm{m\ s^{-1}}$ would move in the confinement time of

$2\times10^{-12}\ \mathrm{s}$ will be

$$d = 1.04\times10^{6}\ \mathrm{m\ s^{-1}} \times 2\times10^{-12}\ \mathrm{s} = 2.08\ \mu\mathrm{m}.$$