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Problem 55.57 (RHK)

Assume that a plasma temperature of 1.3×10^8 K is reached in a laser-fusion device. (a) We have to find the most probable speed of a deuteron at this temperature; (b) and calculate the distance such a deuteron would move in the confinement time of 2×10^{-12} s.

Solution:

(a)

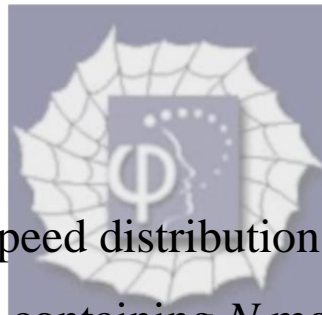
The Maxwellian speed distribution, for a classical gas at temperature T and containing N molecules, is

$$n(v) = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}.$$

The most probable speed is the speed at which $n(v)$ has its maximum value. We find it by requiring that

$$dn(v)/dv = 0.$$

That is



$$\left(2v - v^2 \times \frac{mv}{kT} \right) e^{-mv^2/2kT} = 0,$$

or

$$v_p = \sqrt{\frac{2kT}{m}}.$$

The mass of a deuteron atom will be

$$m_{2\text{H}} = \frac{2.0 \times 10^{-3} \text{ kg}}{6.02 \times 10^{23}} = 3.32 \times 10^{-27} \text{ kg}.$$

Therefore, the most probable speed of a deuteron at $1.3 \times 10^8 \text{ K}$ will be

$$\begin{aligned} (v_{2\text{H}})_p &= \sqrt{\frac{2kT}{m_{2\text{H}}}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 1.3 \times 10^8}{3.32 \times 10^{-27}}} \text{ m s}^{-1} \\ &= 1.04 \times 10^6 \text{ m s}^{-1}. \end{aligned}$$

(b)

The distance a deuteron having a speed of

$1.04 \times 10^6 \text{ m s}^{-1}$ would move in the confinement time of $2 \times 10^{-12} \text{ s}$ will be

$$d = 1.04 \times 10^6 \text{ m s}^{-1} \times 2 \times 10^{-12} \text{ s} = 2.08 \text{ } \mu\text{m}.$$