893.

Problem 55.51 (RHK)

The gravitational potential energy of a uniform spherical object of mass M and radius R is

$$U = -3GM^2/5R,$$

in which G is the gravitational constant. (a) *We have to use this expression to find the maximum energy that could be released by a spherical object, initially of infinite radius, in shrinking to the present size of the Sun* . (b) *Assuming that during this shrinking the Sun radiated Energy at its present rate*; *we have to calculate the age of the Sun based on the hypothesis that the Sun derives its energy from gravitational contraction*.

Solution:

The gravitational potential energy of a uniform spherical object of mass *M* and radius *R* is

$$U = -3GM^2/5R,$$

in which *G* is the gravitational constant. The change in potential energy when the object shrinks from infinite radius to the present radius *R* will be

$$E = \left(U\left(R=\infty\right) - U\left(R\right)\right)$$

$$= 3GM^2/5R = \frac{3\times 6.67\times 10^{-11}\times\left(1.99\times 10^{30}\right)^2}{5\times 6.96\times 10^8}\ \text{J}$$

$$= 2.277\times 10^{41}\ \text{J}.$$

(b)

The Sun is radiating energy at the rate of $3.9\times 10^{26}\ \text{W}$.

The time in which it would have radiated $2.277\times 10^{41}\ \text{J}$ of energy will be

$$t = \frac{2.277\times 10^{41}\ \text{J}}{3.9\times 10^{26}\ \text{W}} = 5.838\times 10^{14}\ \text{s}$$

$$= \frac{5.838\times 10^{14}}{3.156\times 10^7}\ \text{y}$$

$$= 1.85\times 10^7\ \text{y}.$$