891.

Problem 55.49 (RHK)

In certain stars the carbon cycle is more likely than the proton-proton cycle to be effective in generating energy. This cycle is

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma, & Q_1 &= 1.95\ \mathrm{MeV},\\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu, & Q_2 &= 1.19\ \mathrm{MeV},\\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma, & Q_3 &= 7.55\ \mathrm{MeV},\\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma, & Q_4 &= 7.30\ \mathrm{MeV},\\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu, & Q_5 &= 1.73\ \mathrm{MeV},\\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}, & Q_6 &= 4.97\ \mathrm{MeV}.
\end{aligned}
$$

We have to show (a) *that this cycle of reactions is exactly equivalent in its overall effects to the proton-proton cycle*. (b) *We have to verify that this cycle, as expected, has the same* Q *as the* p-p *cycle*.

Solution:

(a) and (b)

The carbon cycle is

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N} + \gamma, \qquad Q_1 = 1.95 \text{ MeV},$$

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^{+} + \nu, \quad Q_2 = 1.19 \text{ MeV},$$

$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + \gamma, \qquad Q_3 = 7.55 \text{ MeV},$$

$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O} + \gamma, \qquad Q_4 = 7.30 \text{ MeV},$$

$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^{+} + \nu, \quad Q_5 = 1.73 \text{ MeV},$$

$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}, \qquad Q_6 = 4.97 \text{ MeV}.$$

We add the six equations and note that effectively we are dealing with the process

$$4\,{}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2e^{+} + 2\nu + 3\gamma.$$

The sum of Qs in the carbon cycle gives

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 = 24.69 \text{ MeV}.$$

The two positrons will readily annihilate with two electrons and their mass energy will contribute to net energy generated by fusion of four protons into one helium nucleus. As the rest mass energy of electron (positron) is 0.51 MeV, the effective Q of the carbon cycle is

$$24.69\ \text{MeV} + 4 \times 0.51\ \text{MeV} = 26.73\ \text{MeV},$$

which is also the Q for the p-p cycle. We note that in the p-p cycle four protons and two electrons get converted into one nucleus of helium and two neutrinos.