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Problem 55.47 (RHK)

Coal burns according to

$$C + O_2 \rightarrow CO_2$$
.

The heat of combustion is $3.3 \times 10^7 \text{ J kg}^{-1}$ of atomic carbon consumed. (a) We have to express this in terms of energy per carbon atom. (b) We have to express it in terms of energy per kilogram of the initial reactants, carbon and oxygen. (c) Suppose that the Sun (mass = $2.0 \times 10^{30} \text{ kg}$) were made of carbon and oxygen in combustible proportions and that it continued to radiate energy at its present rate of $3.9 \times 10^{26} \text{ W}$. We have to find how long it would last.

Solution:

(a)

Coal burns according to

$$C + O_2 \rightarrow CO_2$$
.

The heat of combustion is 3.3×10^7 J kg⁻¹ of atomic carbon consumed. Mass of one carbon atom is 12.000 u.

Therefore, the number of C atoms in 1.0 kg of carbon will be

$$N_{\text{carbon}} = \frac{1.0 \text{ kg}}{12.0 \times 1.6605 \times 10^{-27} \text{ kg}} = 5.02 \times 10^{25}.$$

The heat of combustion of carbon is 3.3×10^7 J kg⁻¹ of atomic carbon consumed. Therefore, energy per carbon atom will be

$$E_{\text{carbon atom}} = \frac{3.3 \times 10^7 \text{ J kg}^{-1}}{5.02 \times 10^{25} \text{ carbon atoms per kg}}$$
$$= 6.573 \times 10^{-19} \text{ J per carbon atom}$$
$$= \frac{6.573 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV per carbon atom}$$
$$= 4.11 \text{ eV per carbon atom.}$$
(b)

The mass of one carbon atom and one O_2 molecule will be 44 u. Therefore, combustion energy per kilogram of carbon and oxygen will be

$$E_{\text{combustion}} = 3.3 \times 10^7 \times \frac{12}{44} \text{ J kg}^{-1}$$
$$= 9.0 \times 10^6 \text{ J kg}^{-1}.$$
(c)

In this model of the Sun, the total energy from combustion of carbon and oxygen that in principle can be obtained will be

$$U = 9.0 \times 10^{6} \times 2 \times 10^{30} \text{ J}$$
$$= 1.8 \times 10^{37} \text{ J}.$$

We assume that the Sun is radiating energy at its present rate of 3.9×10^{26} W. The life of the Sun in this model will therefore be

$$t = \frac{1.8 \times 10^{37} \text{ J}}{3.9 \times 10^{26} \text{ W}} = 4.62 \times 10^{10} \text{ s}$$
$$= \frac{4.62 \times 10^{10}}{3.156 \times 10^{7}} \text{ y}$$
$$= 1462 \text{ y}.$$