

885.

Problem 55.42 (RHK)

At the central core of the Sun the density is $1.5 \times 10^5 \text{ kg m}^{-3}$ and the composition is essentially 35% hydrogen by mass and 65% helium. (a) We have to find the density of protons at the Sun's core. (a) We have to find the ratio of this to the density of particles for an ideal gas at standard conditions of temperature and pressure.



Solution:

(a)

At the central core of the Sun the density is $1.5 \times 10^5 \text{ kg m}^{-3}$ and the composition is essentially 35% hydrogen by mass and 65% helium. Therefore, the contribution to the mass density by hydrogen will be

$$\begin{aligned}\rho_p &= 1.5 \times 10^5 \times 0.35 \text{ kg m}^{-3} \\ &= 0.525 \times 10^5 \text{ kg m}^{-3}.\end{aligned}$$

The mass of proton $m_p = 1.67 \times 10^{-27}$ kg. Therefore, the number density of protons at the central core of the Sun will be

$$n_p = \frac{0.525 \times 10^5 \text{ kg m}^{-3}}{1.67 \times 10^{-27} \text{ kg}} = 3.14 \times 10^{31} \text{ protons per m}^3.$$

(b)

The number density of particles in an ideal gas is determined by its pressure and temperature. The ideal gas equation of state is

$$P = nkT,$$

where k is the Boltzmann constant. The standard conditions of pressure and temperature are

$$P = 1.01 \times 10^5 \text{ Pa}, T = 273 \text{ K}.$$

Therefore,

$$\begin{aligned} n &= \frac{1.01 \times 10^5}{273 \times 1.38 \times 10^{-23}} \text{ particles per m}^3 \\ &= 2.68 \times 10^{25} \text{ particles per m}^3. \end{aligned}$$

And

$$\frac{n_p}{n} = \frac{3.14 \times 10^{31} \text{ protons per m}^3}{2.68 \times 10^{25} \text{ particles per m}^3} = 1.17 \times 10^6.$$