880.

<u>Problem 55.33 (RHK)</u>

Mixed in with $^{238}\mathrm{U}$, *uranium mined today contains* 0.72% *of fissionable* $^{235}\mathrm{U}$, *too little to make reactor fuel for thermal-neutron fission. For this reason, the natural uranium must be enriched with* $^{235}\mathrm{U}$. *Both* $^{235}\mathrm{U}$, $t_{1/2} = 7.0\times10^{8}\ \mathrm{y}$, *and* $^{238}\mathrm{U}$, $t_{1/2} = 4.5\times10^{9}\ \mathrm{y}$, *are radioactive. We have to calculate how far back in time would natural uranium have been a practical reactor fuel, with a* $^{235}\mathrm{U}/^{238}\mathrm{U}$ *ratio of* 3.0%.

<u>Solution:</u>

As the half-life of $^{235}\mathrm{U}$ is 7.0×10^{8} y, its disintegration constant is

$$\lambda_{^{235}\mathrm{U}} = \frac{\ln 2}{7.0\times10^{8}\ \mathrm{y}} = 9.902\times10^{-10}\ \mathrm{y}^{-1},$$

and as the half-life of $^{238}\mathrm{U}$ is $4.5\times10^{9}\ \mathrm{y}$, its disintegration constant is

$$\lambda_{^{238}\mathrm{U}} = \frac{\ln 2}{4.5\times10^{9}\ \mathrm{y}} = 1.540\times10^{-10}\ \mathrm{y}^{-1}.$$

Let the total number of ${}^{235}\mathrm{U}$ at time $t = 0$ be $N_{{}^{235}\mathrm{U}}(0)$ and those of ${}^{238}\mathrm{U}$ then be $N_{{}^{238}\mathrm{U}}(0)$.

Let the number of ${}^{235}\mathrm{U}$, t years later, that is at present, be $N_{{}^{235}\mathrm{U}}(t)$ and those of ${}^{238}\mathrm{U}$, t years later, be $N_{{}^{238}\mathrm{U}}(t)$.

From the radioactive decay law, we note that

$$N_{{}^{235}\mathrm{U}}(t) = N_{{}^{235}\mathrm{U}}(0)\exp\left(-\lambda_{{}^{235}\mathrm{U}} t\right),$$

and

$$N_{{}^{238}\mathrm{U}}(t) = N_{{}^{238}\mathrm{U}}(0)\exp\left(-\lambda_{{}^{238}\mathrm{U}} t\right).$$

We are given that

$$\frac{N_{{}^{235}\mathrm{U}}(t)}{N_{{}^{238}\mathrm{U}}(t)} = 0.0072,$$

and

$$\frac{N_{{}^{235}\mathrm{U}}(0)}{N_{{}^{238}\mathrm{U}}(0)} = 0.03.$$

We thus have the relation

$$\frac{N_{^{235}\mathrm{U}}(t)}{N_{^{238}\mathrm{U}}(t)} = \frac{N_{^{235}\mathrm{U}}(0)}{N_{^{238}\mathrm{U}}(0)} \exp\left(-\left(\lambda_{^{235}\mathrm{U}} - \lambda_{^{238}\mathrm{U}}\right)t\right),$$

or

$$0.0072 = 0.03 \exp\left(-\left(\lambda_{^{235}\mathrm{U}} - \lambda_{^{238}\mathrm{U}}\right)t\right),$$

or

$$\left(\lambda_{^{235}\mathrm{U}} - \lambda_{^{238}\mathrm{U}}\right)t = \ln\left(\frac{0.03}{0.0072}\right) = 1.427,$$

and

$$t = \frac{1.427}{\left(9.902\times10^{-10} - 1.540\times10^{-10}\right)\ \mathrm{y}^{-1}}$$

$$= 1.70\times10^{9}\ \mathrm{y}.$$