875.

Problem 55.28 (RHK)

Among the many fission products that may be extracted chemically from the spent fuel of a nuclear power reactor is $^{90}\mathrm{Sr}\ (t_{1/2} = 29\ \mathrm{y})$. *It is produced in typical large reactors at the rate of* $18\ \mathrm{kg\,y^{-1}}$. *By its radioactivity it generates thermal energy at the rate of* $2.3\ \mathrm{W/g}$. (a) *We have to calculate the effective disintegration energy* Q_{eff} *associated with the decay of a* $^{90}\mathrm{Sr}$ *nucleus.* (Q_{eff} *includes contributions from the decay of* $^{90}\mathrm{Sr}$ *daughter products in its decay chain but not from neutrinos, which escape totally from the sample.*) (b) *It is desired to construct a power source generating* 150 W (*electric*) *to use in operating electronic equipment in an underwater acoustic beacon. If the source is based on the thermal energy generated by* $^{90}\mathrm{Sr}$ *and if the efficiency of the thermal-electric conversion process is* 5.0% , *we have to calculate the amount of* $^{90}\mathrm{Sr}$ *that will be needed. The atomic mass of* $^{90}\mathrm{Sr}$ *is* 89.9 u.

Solution:

(a)

The atomic mass of ${}^{90}\mathrm{Sr}$ is 89.9 u. Therefore, number of ${}^{90}\mathrm{Sr}$ atoms contained in 1.0 g of ${}^{90}\mathrm{Sr}$ will be

$$N = \frac{1.0\ \mathrm{g}}{89.9 \times 1.6605 \times 10^{-24}\ \mathrm{g}} = 6.698 \times 10^{21}.$$

As the half-life of disintegration of ${}^{90}\mathrm{Sr}$ is $t_{1/2} = 29\ \mathrm{y}$, its disintegration constant will be

$$\lambda = \frac{\ln 2}{29 \times 3.156 \times 10^{7}\ \mathrm{s}} = 7.573 \times 10^{-10}\ \mathrm{s}^{-1}.$$

One gram of ${}^{90}\mathrm{Sr}$ will decay at the rate

$$R = \lambda N = 7.573 \times 10^{-10} \times 6.698 \times 10^{21}\ \mathrm{s}^{-1}$$
$$= 50.72 \times 10^{11}\ \mathrm{s}^{-1}.$$

By its radioactivity 1.0 g of ${}^{90}\mathrm{Sr}$ generates thermal energy at the rate of 2.3 J per second. Therefore, the effective disintegration energy Q_{eff} associated with the decay of a ${}^{90}\mathrm{Sr}$ nucleus will be

$$Q_{\mathrm{eff}} R = 2.3\ \mathrm{J\ s}^{-1},$$

or

$$Q_{\mathrm{eff}} = \frac{2.3\ \mathrm{J\ s}^{-1}}{50.72 \times 10^{11}\ \mathrm{s}^{-1}} = 4.53 \times 10^{-13}\ \mathrm{J}$$

$$= \frac{4.53 \times 10^{-13}}{1.6 \times 10^{-13}}\ \mathrm{MeV} = 2.8\ \mathrm{MeV}.$$

(b)

It is desired to construct using $^{90}\mathrm{Sr}$ a power source generating 150 W (electric) to use in operating electronic equipment in an underwater acoustic beacon. It is given that the efficiency of the thermal-electric conversion process is 5.0%. Therefore, the amount of $^{90}\mathrm{Sr}$ required will be

$$2.3 \times 0.05 \times m \text{ W g}^{-1} = 150 \text{ W},$$

or

$$m = \frac{150}{2.3 \times 0.05} \text{ g} = 1.3 \text{ kg}.$$