

871.

Problem 55.22 (RHK)

A neutron with initial kinetic energy K makes a head-on elastic collision with a resting atom of mass m .

(a) We have to show that the fractional energy loss of the neutron is given by

$$\frac{\Delta K}{K} = \frac{4m_n m}{(m + m_n)^2},$$

In which m_n is the neutron mass. (b) We have to find $\Delta K/K$ if the resting atom is hydrogen, deuterium, carbon, or lead. (c) If $K = 1.00$ MeV initially, we have to find how many such collisions would it take to reduce the neutron energy to thermal values (0.025 eV) if the material is deuterium, a commonly used moderator. (Note: In actual moderators, most collisions are not “head-on.”)

Solution:

(a)

Let the speed of the incident neutron be v . We assume that the neutron has a head-on collision with a nuclide of

mass m and it rebounds with speed v' and that the atom of mass m moves forward with speed V after the head-on collision. By applying the principle of conservation of momentum, we write the equation

$$m_n v = mV - m_n v'. \quad (1)$$

As the collision is elastic, conservation of kinetic energy gives us the second algebraic equation,

$$\frac{1}{2} m_n v^2 = \frac{1}{2} m_n v'^2 + \frac{1}{2} mV^2. \quad (2)$$

We eliminate V from equations (1) and (2), and solve for v' . We assume that $m > m_n$, the solution that gives a recoil speed to neutron on head-on collision with an atom of mass m is easily found to be

$$v' = \left(\frac{m - m_n}{m + m_n} \right) v.$$

Therefore, the change in the kinetic energy of the neutron after its head-on collision with an atom of mass m will be

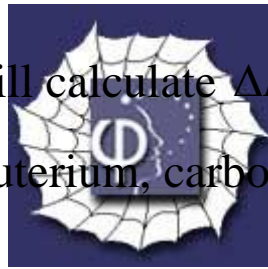
$$\begin{aligned}\Delta K &= \frac{1}{2}m_n v^2 - \frac{1}{2}m_n v'^2 \\ &= \frac{1}{2}m_n v^2 \left(1 - \left(\frac{m - m_n}{m + m_n} \right)^2 \right) \\ &= \frac{1}{2}m_n v^2 \left(\frac{4mm_n}{(m + m_n)^2} \right),\end{aligned}$$

or

$$\frac{\Delta K}{K} = \frac{4mm_n}{(m + m_n)^2}.$$

(b)

In the next part we will calculate $\Delta K/K$ if the resting atom is hydrogen, deuterium, carbon, or lead.



Hydrogen

$$m = m_n,$$

and

$$\frac{\Delta K}{K} = 1.$$

Deuterium

$$m = 2m_n,$$

and

$$\frac{\Delta K}{K} = \frac{8}{9} = 0.88.$$

Carbon

$$m = 12m_n,$$

and

$$\frac{\Delta K}{K} = \frac{48}{(12+1)^2} = 0.28.$$

Lead

$$m = 207m_n,$$

and

$$\frac{\Delta K}{K} = \frac{207}{(207+1)^2} = 0.019.$$

(c)



We have to estimate the number of head-on collisions with deuterium atoms that would reduce the energy of 1.00 MeV neutron to 0.025 eV.

With each collision the kinetic energy of the neutron gets changed from K to $K - \Delta K$, or equivalently by the factor

$$(1 - \alpha), \text{ where } \alpha = \frac{4m_n m}{(m + m_n)^2}.$$

Therefore, after n collisions the energy of the neutron will change to

$$(1 - \alpha)^n K.$$

We have to solve for n given that $K = 1.0 \text{ MeV}$ and

$$(1 - \alpha)^n K = 25 \times 10^{-9} \text{ MeV}.$$

For deuterium atom, $\alpha = 0.888$, and therefore

$$(1 - 0.888)^n = 25 \times 10^{-9},$$

or

$$\begin{aligned} n \log(0.112) &= \log 25 - 9 \\ &= 1.398 - 9, \end{aligned}$$

or

$$n = \frac{7.602}{0.951} = 7.99.$$

That is after about 8 head-on collisions with stationary deuterium atoms, a 1.00 MeV neutron will have its energy reduced to 0.025 eV.

