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Problem 55.22 (RHK)

A neutron with initial kinetic energy K makes a head-on elastic collision with a resting atom of mass m. (a) *We have to show that the fractional energy loss of the neutron is given by*

$$\frac{\Delta K}{K} = \frac{4m_{\mathrm{n}}m}{\left(m + m_{\mathrm{n}}\right)^2},$$

In which m_{n} *is the neutron mass.* (b) *We have to find* $\Delta K/K$ *if the resting atom is hydrogen, deuterium, carbon, or lead.* (c) *If* $K = 1.00\ \mathrm{MeV}$ *initially, we have to find how many such collisions would it take to reduce the neutron energy to thermal values* (0.025 eV) *if the material is deuterium, a commonly used moderator.* (*Note*: *In actual moderators, most collisions are not* "*head-on.*")

Solution:

(a)

Let the speed of the incident neutron be *v*. We assume that the neutron has a head-on collision with a nuclide of mass m and it rebounds with speed v' and that the atom of mass m moves forward with speed V after the head-on collision. By applying the principle of conservation of momentum, we write the equation

$$m_{\mathrm{n}}v = mV - m_{\mathrm{n}}v'. \qquad (1)$$

As the collision is elastic, conservation of kinetic energy gives us the second algebraic equation,

$$\tfrac{1}{2}m_{\mathrm{n}}v^2 = \tfrac{1}{2}m_{\mathrm{n}}v'^2 + \tfrac{1}{2}mV^2. \qquad (2)$$

We eliminate V from equations (1) and (2), and solve for v'. We assume that $m > m_{\mathrm{n}}$, the solution that gives a recoil speed to neutron on head-on collision with an atom of mass m is easily found to be

$$v' = \left(\frac{m - m_{\mathrm{n}}}{m + m_{\mathrm{n}}}\right)v.$$

Therefore, the change in the kinetic energy of the neutron after its head-on collision with an atom of mass m will be

$$\Delta K = \tfrac{1}{2} m_\mathrm{n} v^2 - \tfrac{1}{2} m_\mathrm{n} v'^2$$

$$= \tfrac{1}{2} m_\mathrm{n} v^2 \left(1 - \left(\frac{m - m_\mathrm{n}}{m + m_\mathrm{n}} \right)^2 \right)$$

$$= \tfrac{1}{2} m_\mathrm{n} v^2 \left(\frac{4 m m_\mathrm{n}}{\left(m + m_\mathrm{n}\right)^2} \right),$$

or

$$\frac{\Delta K}{K} = \frac{4 m m_\mathrm{n}}{\left(m + m_\mathrm{n}\right)^2}.$$

(b)

In the next part we will calculate $\Delta K/K$ if the resting atom is hydrogen, deuterium, carbon, or lead.

Hydrogen

$$m = m_\mathrm{n},$$

and

$$\frac{\Delta K}{K} = 1.$$

Deuterium

$$m = 2m_\mathrm{n},$$

and

$$\frac{\Delta K}{K} = \frac{8}{9} = 0.88.$$

Carbon

$m = 12m_{\text{n}}$,

and

$$\frac{\Delta K}{K} = \frac{48}{(12+1)^2} = 0.28.$$

Lead

$m = 207m_{\text{n}}$,

and

$$\frac{\Delta K}{K} = \frac{207}{(207+1)^2} = 0.019.$$

(c)

We have to estimate the number of head-on collisions with deuterium atoms that would reduce the energy of 1.00 MeV neutron to o.025 eV.

With each collision the kinetic energy of the neutron gets changed from K to $K - \Delta K$, or equivalently by the factor

$$(1-\alpha), \text{ where } \alpha = \frac{4m_{\text{n}}m}{(m+m_{\text{n}})^2}.$$

Therefore, after *n* collisions the energy of the neutron will change to

$$(1-\alpha)^n K.$$

We have to solve for n given that $K = 1.0$ MeV and

$(1-\alpha)^n K = 25\times10^{-9}$ MeV.

For deuterium atom, $\alpha = 0.888$, and therefore

$$(1-0.888)^n = 25\times10^{-9},$$

or

$$n\log(0.112) = \log 25 - 9$$
$$= 1.398 - 9,$$

or

$$n = \frac{7.602}{0.951} = 7.99.$$

That is after about 8 head-on collisions with stationary deuterium atoms, a 1.00 MeV neutron will have its energy reduced to 0.025 eV.