869.

Problem 55.18 (RHK)

A $^{236}\mathrm{U}^*$ *nucleus undergoes fission and breaks up into two middle-mass fragments,* $^{140}\mathrm{Xe}$ *and* $^{94}\mathrm{Sr}$. *We have to answer the following*: (a) *By what percentage the does the surface are of the* $^{236}\mathrm{U}^*$ *nucleus change during the process*? (a) *By what percentage does its volume change*? (c) *By what percentage does its electrostatic potential energy change? The electrostatic potential energy of a uniformly charged sphere of radius r and charge Q is given by*

$$U = \frac{3}{5}\left(\frac{Q^2}{4\pi\varepsilon_0 r}\right).$$

Solution:

We will calculate the radii of $^{140}\mathrm{Xe}$ and $^{94}\mathrm{Sr}$ nuclides using the empirical relation

$$r = r_0 A^{1/3},$$

$$r_0 = 1.2\times 10^{-15}\ \text{m}.$$

The radius of the $^{236}\mathrm{U}^*$ nucleus will therefore be

$$r_{^{236}\mathrm{U}^*} = 1.2 \times (236)^{1/3} \times 10^{-15}\ \mathrm{m}$$
$$= 7.41 \times 10^{-15}\ \mathrm{m} = 7.41\ \mathrm{fm}.$$

The radius of the $^{140}\mathrm{Xe}$ nuclide will be

$$r_{^{140}\mathrm{Xe}} = 1.2 \times (140)^{1/3} \times 10^{-15}\ \mathrm{m}$$
$$= 6.23 \times 10^{-15}\ \mathrm{m} = 6.23\ \mathrm{fm}.$$

And, the radius of the $^{94}\mathrm{Sr}$ nuclide will be

$$r_{^{94}\mathrm{Sr}} = 1.2 \times (94)^{1/3} \times 10^{-15}\ \mathrm{m}$$
$$= 5.46 \times 10^{-15}\ \mathrm{m} = 5.46\ \mathrm{fm}.$$

The atomic number of U nuclide is 92, that of $^{140}\mathrm{Xe}$ nuclide is 54, and the atomic number of $^{94}\mathrm{Sr}$ nuclide is 38. Therefore, the charge contained in a $^{236}\mathrm{U}^*$ nucleus is 92*e*, the charge contained in a $^{140}\mathrm{Xe}$ nucleus is 54*e*, and the charge contained in a $^{94}\mathrm{Sr}$ nucleus is 38*e*.

(a)

The surface area of the $^{236}\mathrm{U}^*$ nucleus will therefore be

$$A_{^{236}\mathrm{U}^*} = 4\pi r^2_{^{236}\mathrm{U}^*} = 4\pi\left(7.41^2\right)\ \mathrm{fm}^2$$
$$= 4\pi(54.9)\ \mathrm{fm}^2.$$

The total surface area of one $^{140}\mathrm{Xe}$ nucleus and one $^{94}\mathrm{Sr}$ nucleus will be

$$A_{^{140}\text{Xe} + ^{94}\text{Sr}} = 4\pi\left(r^2_{^{140}\text{Xe}} + r^2_{^{94}\text{Sr}}\right) = 4\pi\left(6.23^2 + 5.46^2\right)\ \text{fm}^2$$
$$= 4\pi\left(68.6\right)\ \text{fm}^2.$$

The percentage change in the area of the $^{236}\text{U}^*$ nucleus when it fissions into one nucleus of ^{140}Xe and one nucleus of ^{94}Sr will therefore be

$$\frac{A_{^{140}\text{Xe} + ^{94}\text{Sr}} - A_{^{236}\text{U}^*}}{A_{^{236}\text{U}^*}} = \frac{68.6 - 54.9}{54.9} = 0.249,$$

or

24.9%.

(b)

The volume of the $^{236}\text{U}^*$ nucleus will therefore be

$$V_{^{236}\text{U}^*} = \frac{4\pi}{3} r^3_{^{236}\text{U}^*} = \frac{4\pi}{3} r_0^3 \times \left(236\right)$$
$$= \frac{4\pi}{3}\left(236\right) r_0^3.$$

The total volume of one ^{140}Xe nucleus and one ^{94}Sr nucleus will be

$$V_{^{140}\text{Xe} + ^{94}\text{Sr}} = \frac{4\pi}{3}\left(r^3_{^{140}\text{Xe}} + r^3_{^{94}\text{Sr}}\right) = \frac{4\pi}{3} r_0^3\left(140 + 94\right)$$
$$= \frac{4\pi}{3}\left(234\right) r_0^3.$$

And, the percentage change in the volume of the $^{236}\mathrm{U}^*$ nucleus when it fissions into one nucleus of $^{140}\mathrm{Xe}$ and one nucleus of $^{94}\mathrm{Sr}$ will therefore be

$$\frac{V_{^{140}\mathrm{Xe}+^{94}\mathrm{Sr}} - V_{^{236}\mathrm{U}^*}}{V_{^{236}\mathrm{U}^*}} = \frac{-2}{236} = -8.47\times10^{-3},$$

or

–0.85%.

(c)

The electrostatic potential energy of a uniformly charged sphere of radius r and charge Q is given by

$$U = \frac{3}{5}\left(\frac{Q^2}{4\pi\varepsilon_0 r}\right).$$

The electrostatic potential energy of a $^{236}\mathrm{U}^*$ nucleus will be

$$U_{^{236}\mathrm{U}^*} = \frac{3}{5\times(4\pi\varepsilon_0)}\times\frac{Q^2_{^{236}\mathrm{U}^*}}{r_{^{236}\mathrm{U}^*}} = \frac{3}{5\times(4\pi\varepsilon_0)}\times\frac{(92e)^2}{7.41\ \mathrm{fm}}$$

$$= \frac{3}{5\times(4\pi\varepsilon_0)}\times 1142e^2\mathrm{fm}^{-1}.$$

The total electrostatic potential energy of one ^{140}Xe nucleus and one ^{94}Sr nucleus will be

$$U_{^{140}\text{Xe}} + U_{^{94}\text{Sr}} = \frac{3}{5\times(4\pi\varepsilon_0)} \times \left(\frac{Q^2_{^{140}\text{Xe}}}{r_{^{140}\text{Xe}}} + \frac{Q^2_{^{94}\text{Sr}}}{r_{^{94}\text{Sr}}} \right)$$

$$= \frac{3}{5\times(4\pi\varepsilon_0)} \times \left(\frac{(54e)^2}{6.23\ \text{fm}} + \frac{(38e)^2}{5.46\ \text{fm}} \right)$$

$$= \frac{3}{5\times(4\pi\varepsilon_0)} \times \mathbf{732}\ e^2\text{fm}^{-1}.$$

The percentage change in the potential energy of the $^{236}\text{U}^*$ nucleus when it fissions into one nucleus of ^{140}Xe and one nucleus of ^{94}Sr will therefore be

$$\frac{U_{^{140}\text{Xe}} + U_{^{94}\text{Sr}} - U_{^{236}\text{U}^*}}{U_{^{236}\text{U}^*}} = \frac{\mathbf{732} - 1142}{1142},$$

or

-36%